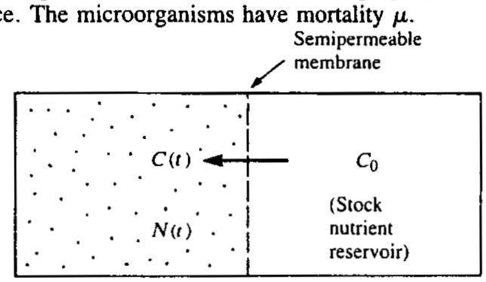
7. Michaelis-Menten kinetics were selected for the nutrient-dependent bacterial growth rate in Section 4.4.
   (a) Show that if $K(C)$ is given by equation (15) a half-maximal growth rate is attained when the nutrient concentration is $C = K_n$.
   (b) Suppose instead we assume that $K(C) = K_m C$, where $K_m$ is a constant. How would this change the steady state $(\bar{N}_1, \bar{C}_1)$?
   (c) Determine whether the steady state found in part (b) would be stable.

11. In industrial applications, one wants not only to ensure that the steady state $(\bar{N}_1, \bar{C}_1)$ given by equations (25a, b) exists, but also to increase the yield of bacteria, $\bar{N}_1$. Given that one can in principle adjust such parameters as $V$, $F$, and $C_0$, how could this be done?

12. In this question we deal with a number of variants of the chemostat.
   (a) How would you expect the model to differ if there were two growth-limiting nutrients?
   (b) Suppose that at high densities bacteria start secreting a chemical that inhibits their own growth. How would you model this situation?
   (c) In certain cases two (or more) bacterial species are kept in the same chemostat and compete for a common nutrient. Suggest a model for such competition experiments.

13. Consider equation (8) for the growth of a microorganism in a nutrient-limiting environment.
   (a) By making appropriate choices for units of measurement $\hat{N}$ and $\tau$ (for time), bring the equation to dimensionless form.
   (b) What are the steady states of the equation?
   (c) Determine the stability of these steady states by linearizing the equation about the steady states obtained in (b).
   (d) Verify that your results agree with the exact solution given by equation (10).

14. In the hypothetical growth chamber shown here, the microorganisms (density $N(t)$) and their food supply are kept in a chamber separated by a semipermeable membrane from a reservoir containing the stock nutrient whose concentration $[C_0 > C(t)]$ is assumed to be fixed. Nutrient can pass across the membrane by a process of diffusion at a rate proportional to the concentration difference. The microorganisms have mortality $\mu$.




   (a) Explain the following equations:

$$\frac{dN}{dt} = N\left(\frac{K_{\max} C}{K_n + C}\right) - \mu N, \qquad \frac{dC}{dt} = D[C_0 - C(t)] - \alpha N \frac{K_{\max} C}{K_n + C}.$$

   (d) Find all steady states.
   (e) Carry out a stability analysis and find the constraints that parameters must satisfy to ensure stability of the nontrivial steady state.