(Problems pasted from Keshet's book.)

(1) Determine if Bendixson's criterion can be used to rule out periodic orbits in the following systems, in the indicated regions of the plane:

(a) $\dfrac{dx}{dt} = 2x - 3y,$ $\dfrac{dy}{dt} = 10x + y,$ (in R^2).

(b) $\dfrac{dx}{dt} = x + 2y(\sin^2 y) + y^{1/2},$ $\dfrac{dy}{dt} = xe^x + \dfrac{x^2}{2} + y,$ (in R^2).

(c) $\dfrac{dx}{dt} = a_1x + b_1y + c_1x^2,$ $\dfrac{dy}{dt} = d_2x + b_2y + c_2y^2,$ $(x > 0,\ y > 0)$.

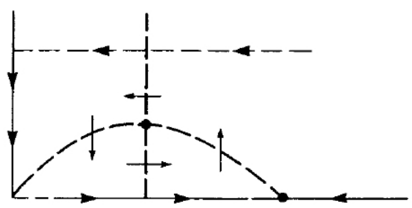
(d) $\dfrac{dx}{dt} = ax - bxy,$ $\dfrac{dy}{dt} = -cy + dxy,$ $(x > 0,\ y > 0)$.

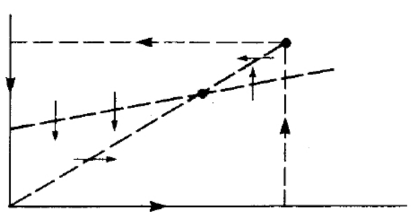
(2) Suppose that all the steady states (indicated by large dots) in the figures shown below are repelling.

Can the Poincare-Bendixson Theorem be applied to conclude the existence of limit cycles?

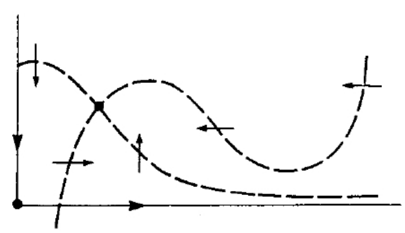
(You have to decide if there are trapping regions.)

(a)





(b)



(c)

(3)

A vector field and cubic nullclines in a model for respiration in a bacterial culture. The nutrient x and oxygen y are assumed to satisfy the equations

$$\frac{dx}{dt} = B - x - \frac{xy}{1 + qx^2} = F(x, y),$$

$$\frac{dy}{dt} = A - \frac{xy}{1 + qx^2} = G(x, y).$$

Typical parameter values for stable oscillations are

$A = 11.0,\ B = 19.4,\ q = 0.5.$

[From Fairen and Velarde (1979). Fig. 3, p. 152, J. Math. Biol., 8, 147–157, by permission of Springer-Verlag.]

Use a computer to show that, for these parameters, the phase-plane looks like: