Math 338, Problem and Reading Assignments, Spring 2008

Week 8

Note the exercises starting after 4.24 in the text are misnumbered. The second exercise on page 39 should be 4.25, the exercise after that 4.26, etc. The numbers below should be interpreted with this correction. Basically the homework ask you to do all remaining problems in Chapter 4, starting with 4.24.

(You may read about moment generating functions in Chapter 2.)

1. Exercise 4.24, page 39.

2. Exercise 4.25, page 39.

3. Exercise 4.26, page 40.

4. Exercise 4.27, page 45.

5. Exercise 4.28, page 45.

6. Exercise 4.29, page 45

7. The moment generating function for normal random variables is given in the table on page 34 of Chapter 2.

a) Use the moment generating function to show that the sum of independent normal random variables is normal.

b) Derive the formula for the moment generating function of the normal random variable.

8. This problem helps see why the Theorem on page 36 about Poisson thinning is true. Let Z be a Poisson random variable with mean λ. For concretenss, imagine Z represents a random number of arrivals to a service desk in an hour. Each arrival is accepted with probability μ or rejected with probability $1-\mu$, independently of each other. Let Y denote the total number of accepted arrivals. The object of this problem is to show that Y is a Poisson random variable with mean $\mu\lambda$.

To get started on this problem, it helps to have a more precise definition of the pair (Y, Z). The description we gave above is really equivalent to the following: Z has a Poisson distribution with parameter λ and, for every k, the conditional distribution of Y given $Z = k$ is binomial with parameters k and μ. The reason is that, given $Z = k$, the number of arrivals is determined by k independent trials with the probability of success on each trial being μ. Mathematically,

$$\mathbb{P}\left(Y=j \mid Z=k\right) = \binom{k}{j}\mu^j(1-\mu)^{n-j}$$

For this problem we recall the fundamental identity for expectations: given two random variables, X and W, where W is discrete,

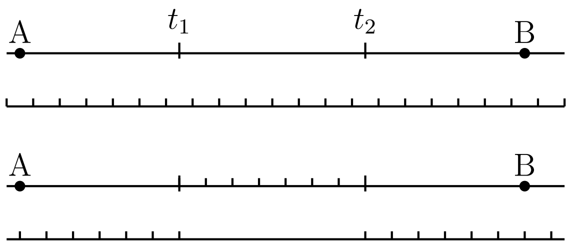
$$E[X] = \sum_k E[X \mid W=k]\,\mathbb{P}(W=k).$$

a) Calculate $E[Y \mid Z=k]$ for any integer k. You can do this with practically no work if you use the table on page 34 of chapter 2 that supplies moment generating functions of basic random variables.

b) Compute $E\left[e^{tY}\right]$. Use (a) and the formula above for computing an expectation, taking X to be e^{tY} and W to be Z.

c) Conclude that Y is Poisson with parameter $\mu\lambda$.

9. (Source: Karlin and Taylor, *Introduction to Stochastic Processes*.) In meiosis, breaks occur along a pair of chromosomes and they recombine so that a piece of chromosome is exchanged. The following figure is a before-and-after picture in which the breaks occur at points t_1 and t_2 and the middle section is exchanged.



Suppose that breaks occur along the chromosome according to a Poisson process.

a) What kind of assumptions about breaks would suggest this model is a good approximation?

b) Assume that the location of breaks is a Poisson process with rate λ. Let the distance between A and B be called ℓ. Find an expression for the probability that sites A and B remain on the same chromosome after recombination.