

Math 338, Additional Mutation Problem for Chapter 3

Consider a single locus admitting 3 alleles, A , B , and C . Assume infinite population, non-overlapping generations, random mating, monocious organism, no selection or migration. Suppose in mating A mutates to B with probability $1/4$, B mutates to C with probability $1/3$ and C mutates to A with probability $1/3$.

a) Derive a system of 3, coupled difference equations that express, respectively, $f_A(t+1)$, $f_B(t+1)$, and $f_C(t+1)$ in terms of $f_A(t)$, $f_B(t)$, $f_C(t)$. Then reduce this to a system of equations for f_A and f_B only by using $f_A(t) + f_B(t) + f_C(t) = 1$.

b) Write the iteration for f_A and f_B as a non-homogeneous matrix iteration of the form:

$$x(t+1) = Ax(t) + b$$

where $x(t)$ is a column vector consisting of $f_A(t)$ and $f_B(t)$, A is a 2 by 2 matrix, and b is a vector.

Show that the eigenvalues of A are approximately $0.5417 \pm 0.2602i$ (use a calculator, or MATLAB, Maple, etc).

Show that these eigenvalues have magnitude less than 1.

c) Show that the only fixed point of this iteration (i.e., solve $Ax + b = x$) is $x = 0.4$, $y = 0.3$.

We conclude (as will be explained in class) that, no matter what the initial values of $f_A(0)$, $f_B(0)$ and $f_C(0)$ are, $\lim_{t \rightarrow \infty} f_A(t) = 0.4$, $\lim_{t \rightarrow \infty} f_B(t) = 0.3$, $\lim_{t \rightarrow \infty} f_C(t) = 1 - 0.4 - 0.3 = 0.3$.