Proceedings of the 42nd IEEE
Conference on Decision and Control
Maui, Hawaii USA, December 2003

Rendering the Electromechanical Valve Actuator Globally Asymptotically Stable

Katherine S. Peterson and Anna G. Stefanopoulou*
Mechanical Engineering Department
University of Michigan
Ann Arbor, MI 48109-4256
kspeters@engin.umich.edu and annastef@umich.edu

***Abstract*— This paper presents a nonlinear controller based on Sontag's feedback to render the electromechanical valve actuator (EVA) globally asymptotically stable (GAS). Electromechanical valve actuators have received much attention recently due to their potential for improving the performance of the internal combustion engine. Various control schemes for the EVA have been proposed, however stability is often neglected due to the bounded motion of the EVA or proven based on a linearized plant model. Here, we demonstrate and prove that our controller renders the system GAS without any assumption of linearity.**

I. INTRODUCTION

Variable valve timing (VVT) can significantly improve the performance of the internal combustion engine by decoupling the engine valve timing from other engine events. This flexibility allows for better engine performance which can potentially reduce automotive emissions by 12% to 15% [13], improve torque output by 20% [3], and reduce fuel consumption by 18% to 23% [7].

Devices capable of achieving VVT range from switching between multiple cam profiles [9], individual cam drives [4], piezo's [17], and hydraulic systems [8]. Electromechanical valve actuators (EVA) which use electromagnets to actuate the engine valves are becoming increasingly popular due to their ability to achieve continuously variable lift, duration, and phasing of the valves. Unfortunately, the EVA suffers from large impacts at various internal locations. These impacts must be kept under 0.1 m/s as they are otherwise excessively loud and damaging to the actuator and engine valve.

Controllers that eliminate or reduce these impacts often neglect stability as the motion of the EVA is bounded by physical constraints [1], [10]. Those which do prove stability [5], [11], [14], [15] do so based on a linearized plant model, thus only proving stability locally about the equilibrium point. The resulting stability is never shown experimentally as the moving components of the EVA are only ever brought to rest against physical constraints which is achievable with open loop control.

This paper presents a nonlinear controller based on Sontag's feedback which renders the electromechanical valve actuator globally asymptotically stable (GAS). This result is proven mathematically and shown experimentally by stabilizing the system away from the physical constraints. Additionally, a methodology for achieving the desired performance is given.

*Support is provided by the National Science Foundation under contract NSF-ECS-0049025 and Ford Motor Company through a 2001 University Research Project.

II. THE ELECTROMECHANICAL VALVE ACTUATOR

The electromechanical valve actuator and experimental setup are shown in Fig. 1. The actuator consists of an armature mounted between two opposing magnetic coils and springs. The experimental setup consists of a 200 V power supply, two Pulse Width Modulated (PWM) drivers, an eddy current sensor mounted on the rear of the actuator, a laser vibrometer, and a 1103 dSpace processing board. During operation the dSpace processing board regulates the duty cycle command to the PWM drivers in order to govern the motion of the armature. The commands from the dSpace processing board are based on either open loop instructions or measurements from the various sensors. The position of the armature/valve is measured by an eddy current sensor that detects changes in the magnetic field due to the motion of the sensor target. The velocity of the armature/valve is measured by a laser vibrometer. Lastly, the current in each electromagnet is measured by sensors build into each PWM driver. The controller, which is presented in Sec. III, is implemented using the eddy current sensor and the sensors bundled with the PWM drivers. The laser vibrometer is used to verify the system performance for the experimental results.

At the beginning of a valve opening/closing procedure the armature is held in contact with one of the two magnetic coils, referred to as the releasing coil, creating a force imbalance between the two springs. The current in the releasing coil is reduced to zero and the springs drive the armature across the gap, where it is then caught and held in place by the opposing magnetic coil, referred to as the catching coil. The forcing of the armature between the two extreme positions thereby opens or shuts the valve. If the voltage to the catching coil is not carefully regulated large impacts can occur between it and the armature. These impacts must be kept below 0.1 m/s to avoid damage and excessively loud operation.

In modeling the EVA [2], [16] the effects of the releasing coil are neglected as only the catching coil is used to stabilize the armature. Furthermore, the current in the releasing coil is reduced to zero very rapidly using the technique described in [16].

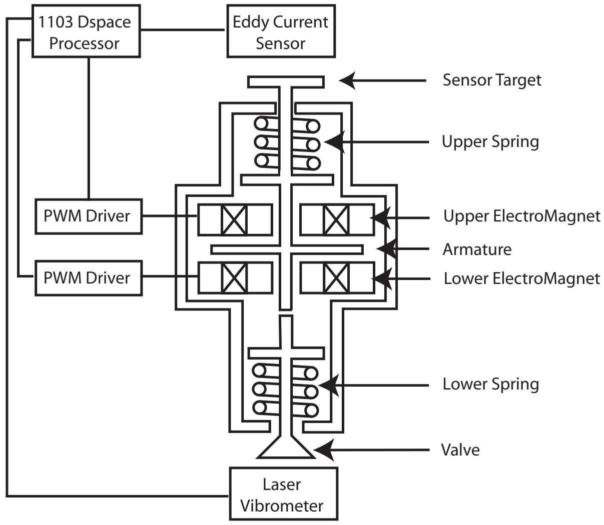

Fig. 1. Electromechanical valve actuator and experimental setup.

The states are defined as:

- z the distance between the armature and catching coil [m].
- v the armature velocity [m/s].
- λ the magnetic flux in the catching coil $\left[A/\left(\sqrt{N}m\right)\right]$.

The dynamics of the electromechanical valve actuator are described by

$$\frac{dz}{dt} = v \tag{1}$$

$$\frac{dv}{dt} = \frac{1}{m}\left(-\frac{\lambda^2}{4k_a} + k_s\left(l - z\right) - bv\right) \tag{2}$$

$$\frac{d\lambda}{dt} = V_c - \frac{\lambda\left(k_b + z\right)r}{2k_a} \tag{3}$$

where m is the combined mass of the armature and valve in kg, $\frac{\lambda^2}{4k_a}$ is the magnetic force generated by the catching coil in N, k_s is the spring constant in N/m, l is half the total armature travel in m, b is the damping coefficient in kg/s, V_c is the applied voltage in V, r is the resistance of both the wiring and magnetic coil in Ω, and k_a and k_b are constants determined by the physical properties of the EVA. Here we assume that the frequency of the PWM drivers are sufficiently fast that the applied voltage is equivalent to the commanded duty cycle multiplied by the supply voltage, which has been set to 180 V.

III. Rendering the EVA Globally Asymptotically Stable

This section presents a controller capable of rending the electromechanical valve actuator GAS. First the state space representation of the system is translated such that the equilibrium is located at the origin. Next we propose a Lyapunov function and prove that it satisfies the condition for a Control Lyapunov function. Finally, the system is rendered GAS using Sontag's feedback.

A. Translated State Space Representation

By defining the new set of coordinates $x = \left[\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}\right]^T$ and input u as

$$x_1 = z - z_{eq} \tag{4}$$

$$x_2 = v - v_{eq} \tag{5}$$

$$x_3 = \lambda - \lambda_{eq} \tag{6}$$

$$u = V_c - V_{eq} \tag{7}$$

where z_{eq}, v_{eq}, and λ_{eq} are the equilibrium values of the states corresponding to the input V_{eq}, the equilibrium is translated so that it resides at the origin. In the derivation which follows it is assumed that z_{eq}, v_{eq}, λ_{eq}, and V_{eq} are not necessarily constant and are once differentiable.

The new state space representation is derived by differentiating (4) (5) (6) with respect to time. Beginning with (4) we get:

$$\frac{dx_1}{dt} = v - \frac{dz_{eq}}{dt} = x_2 + v_{eq} - \frac{dz_{eq}}{dt}. \tag{8}$$

For feasibility, the constraint $\frac{dz_{eq}}{dt} = v_{eq}$ must be satisfied yielding

$$\frac{dx_1}{dt} = x_2. \tag{9}$$

Next, the rate of change of x_2 is given by

$$\frac{dx_2}{dt} = \frac{1}{m}\left(-\frac{\left(x_3 + \lambda_{eq}\right)^2}{4k_a} + k_s\left(l - x_1 - z_{eq}\right) - b\left(x_2 + v_{eq}\right) - m\frac{dv_{eq}}{dt}\right).$$

Let

$$\lambda_{eq} = 2\sqrt{k_a\left(k_s\left(l - z_{eq}\right) - bv_{eq} - m\frac{dv_{eq}}{dt}\right)} \tag{10}$$

resulting in

$$\frac{dx_2}{dt} = \frac{1}{m}\left(-\frac{x_3^2}{4k_a} - k_s x_1 - bx_2\right) - \frac{x_3\lambda_{eq}}{2k_a m}. \tag{11}$$

Finally, the dynamics of the state x_3 are described by

$$\frac{dx_3}{dt} = V_c - \frac{\left(x_3 + \lambda_{eq}\right)\left(k_b + x_1 + z_{eq}\right)r}{2k_a} - \frac{d\lambda_{eq}}{dt}. \tag{12}$$

Defining V_{eq} as

$$V_{eq} = \frac{d\lambda_{eq}}{dt} + \frac{\lambda_{eq}(k_b + z_{eq}) r}{2k_a} \quad (13)$$

results in

$$\frac{dx_3}{dt} = u - \frac{x_3 (k_b + x_1 + z_{eq}) r}{2k_a} - \frac{x_1 \lambda_{eq} r}{2k_a}. \quad (14)$$

Therefore the new state space representation can be expressed in the form $\frac{dx}{dt} = f(x) + g(x) u$ as

$$\frac{dx}{dt} = \underbrace{\begin{bmatrix} x_2 \\ \frac{1}{m}\left(-\frac{x_3^2}{4k_a} - k_s x_1 - b x_2\right) - \frac{x_3 \lambda_{eq}}{2k_a m} \\ -\frac{x_3(k_b+x_1+z_{eq})r}{2k_a} - \frac{x_1 \lambda_{eq} r}{2k_a} \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{g(x)} u$$

B. Proposed Control Lyapunov Function

As this new state space representation is valid for all values of $\begin{bmatrix} z_{eq} & v_{eq} & \lambda_{eq} \end{bmatrix}$ without loss of generality we can stabilize the system about any generic equilibrium point $\begin{bmatrix} z_{eq} & v_{eq} & \lambda_{eq} \end{bmatrix}$. As such, a Lyapunov function is proposed here and shown to satisfy the conditions of a Control Lyapunov Function (CLF). Sontag's feedback is then used to render the system GAS.

We propose the Lyapunov function

$$V = \begin{bmatrix} x_1 & x_2 \end{bmatrix} P \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T + \gamma x_3^2 \quad (15)$$

as a candidate CLF, where γ is constant and positive and the matrix P satisfies the equation

$$A^T P + PA + I = 0 \quad (16)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -k_s/m & -b/m \end{bmatrix}. \quad (17)$$

Since the eigenvalues of A are all strictly less than zero the matrix P is positive definite.

Therefore (15) satisfies the conditions

$$V \geq 0 \qquad \forall x \in R^3 \quad (18)$$
$$V = 0 \qquad \text{iff } x = 0 \quad (19)$$
$$V \to \infty \qquad \text{as } ||x|| \to \infty. \quad (20)$$

In order to be a Control Lyapunov function it must also satisfy the additional constraints

$$LfV \leq 0 \quad \forall x \in R^3 \quad \text{when } LgV = 0 \quad (21)$$
$$LfV = 0 \quad \text{iff } x = 0 \quad (22)$$

where LfV and LgV are the Lie derivatives of V with respect to $f(x)$ and $g(x)$ defined as

$$LfV = \frac{dV}{dx} f(x) \text{ and } LgV = \frac{dV}{dx} g(x). \quad (23)$$

Solving for LgV

$$LgV = \begin{bmatrix} \frac{dV}{dx_1} & \frac{dV}{dx_2} & \frac{dV}{dx_3} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \quad (24)$$
$$LgV = 2\gamma x_3 \quad (25)$$

Therefore

$$LgV = 0 \quad \text{iff} \quad x_3 = 0. \quad (26)$$

When $x_3 = 0$, LfV is given as

$$\begin{aligned} LfV &= \Big(\begin{bmatrix} \frac{dx_1}{dt} & \frac{dx_2}{dt} \end{bmatrix} P \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \\ &\quad + \begin{bmatrix} x_1 & x_2 \end{bmatrix} P \begin{bmatrix} \frac{dx_1}{dt} & \frac{dx_2}{dt} \end{bmatrix}^T\Big) \\ LfV &= \Big(\begin{bmatrix} x_1 & x_2 \end{bmatrix} A^T P \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \\ &\quad + \begin{bmatrix} x_1 & x_2 \end{bmatrix} PA \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T\Big) \\ LfV &= -\begin{bmatrix} x_1 & x_2 \end{bmatrix} I \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T, \end{aligned}$$

which satisfies both (21) (22). Thus (15) is a Control Lyapunov function.

C. Sontag's Feedback

We briefly show here that Sontag's feedback does indeed render the origin GAS. For more information on Lyapunov stability and Sontag's feedback the reader is refereed to [6], [12].

As (15) satisfies the constraints given in (18) (19) (20) (21) (22) application of Sontag's feedback

$$u = \begin{cases} -\frac{LfV + \sqrt{LfV^2 + LgV^4}}{LgV} & \text{for} \quad LgV \neq 0 \\ 0 & \text{for} \quad LgV = 0 \end{cases}. \quad (27)$$

will render the system globally asymptotically stable about the origin.

Taking the derivative of V with respect to time and applying the chain rule

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} = \frac{\partial V}{\partial x}(f(x) + g(x) u) \quad (28)$$
$$\frac{dV}{dt} = LfV + LgV u. \quad (29)$$

When $LgV = 0$, $\frac{dV}{dt}$ is given by

$$\frac{dV}{dt} = LfV. \quad (30)$$

When $LgV \neq 0$, $\frac{dV}{dt}$ is given by

$$\frac{dV}{dt} = LfV + LgVu \quad (31)$$
$$\frac{dV}{dt} = LfV - LgV\frac{LfV + \sqrt{LfV^2 + LgV^4}}{LgV} \quad (32)$$
$$\frac{dV}{dt} = -\sqrt{LfV^2 + LgV^4}. \quad (33)$$

Where by construction both (30) and (33) satisfy the conditions

$$\frac{dV}{dt} \leq 0 \qquad \forall x \in R^3 \quad (34)$$
$$\frac{dV}{dt} = 0 \qquad \text{iff } x = 0. \quad (35)$$

Therefore the system is GAS about the origin.

IV. Equilibrium Selection

In the preceding section the electromechanical valve actuator was rendered globally asymptotically stable for any generic equilibrium set that is once differentiable. However no mention was made of what a practical equilibrium set might be. Since Sontag's feedback only guarantees stability and not performance, we hope to achieve the desired performance of the EVA by careful selection of the equilibrium set. Recall that the desired final state is the armature at rest against the catching magnetic coil. Unfortunately use of this equilibrium set may lead to actuator saturation and degraded performance as the magnetic force is significantly less than the spring force for distances greater than approximately 1 mm away from the catching coil. As the armature nears the catching coil the magnetic flux increases significantly. The application of large control inputs when the armature is not near the catching coil will therefore result in large impact velocities, which is exactly what we wish to avoid.

This approach fails to take into account that the armature is initially held against the opposing magnetic coil compressing the corresponding spring. The potential energy stored in the spring does the majority of the work in driving the armature across the gap. The controller need only provide slight compensation during the majority of the armature travel, and then stabilize it against the magnetic coil at the end of the transition.

Therefore the controller is implemented with two different sets of equilibria. The first set is based on the tracking of a perfect cosine wave. A cosine wave is selected as it represents the free motion of the armature without the effects of damping. As the armature trajectory approaches a cosine wave the impact velocity will tend toward zero, i.e. the velocity of armature becomes a sin-wave with zero terminal velocity. Therefore the equilibrium voltage, V_{eq}, acts as a pseudo feed-forward term to help guide the armature along the desired trajectory, and the controller compensates for deviations captured in the states $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$. The second set corresponds to the armature at rest against the magnetic coil so that the armature ends in the desired final state.

A. 1^{st} Equilibrium Set

For the first set of equilibria, $\begin{bmatrix} z_{eq} & v_{eq} & \lambda_{eq} \end{bmatrix}$, let

$$z_{eq} = l + l\cos(\omega t) \quad (36)$$

where $\omega = \sqrt{\frac{k_s}{m}}$ is the natural frequency of the system. Thus v_{eq} is given by

$$v_{eq} = -l\omega\sin(\omega t). \quad (37)$$

Using (10) (36) (37), λ_{eq} is given by

$$\lambda_{eq} = 2\sqrt{k_a\left(-lm\omega^2\cos(\omega t) - bv_{eq} - m\frac{dv_{eq}}{dt}\right)}, \quad (38)$$

noting that

$$\frac{dv_{eq}}{dt} = -l\omega^2\cos(\omega t) \quad (39)$$

yields

$$\lambda_{eq} = 2\sqrt{bk_a l\omega\sin(\omega t)}. \quad (40)$$

Solving for $\frac{d\lambda_{eq}}{dt}$

$$\frac{d\lambda_{eq}}{dt} = \frac{2lbk_a\omega^2\cos(\omega t)}{\lambda_{eq}} \quad (41)$$

we find

$$V_{eq} = \frac{2lbk_a\omega^2\cos(\omega t)}{\lambda_{eq}} + \frac{\lambda_{eq}(k_b + z_{eq})r}{2k_a}. \quad (42)$$

V_{eq} corresponds to the feed forward voltage applied to the EVA to improve tracking of the desired trajectory (36). The controller based on Sontag's Feedback designed in Sec. III then compensates for deviations captured by the states $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$.

B. 2^{nd} Equilibrium Set

For the second set of equilibria, $\begin{bmatrix} z_{eq} & v_{eq} & \lambda_{eq} \end{bmatrix}$, let

$$z_{eq} = 0 \quad (43)$$
$$v_{eq} = 0 \quad (44)$$
$$\lambda_{eq} = 2\sqrt{k_s k_a l} \quad (45)$$
$$V_{eq} = \frac{\lambda_{eq}k_b r}{2k_a} \quad (46)$$

which corresponds to the armature at rest against the magnetic coil.

C. Switching Between the Equilibria Sets

To achieve the desired performance our controller uses the equilibrium set defined by (36) (37) (38) and feed forward voltage

$$V_{eq} = \begin{cases} 0 & \text{if } \lambda_{eq} = 0 \\ \frac{2lbk_a\omega^2\cos(\omega t)}{\lambda_{eq}} + \frac{\lambda_{eq}(k_b+z_{eq})r}{2k_a} & \text{if } \lambda_{eq} \neq 0 \end{cases} \quad (47)$$

when $t < \pi/\omega$ and $z > 1$ mm. Otherwise the second set of equilibria defined by (43) (44) (45) (46) is used. It is important to note that once the controller switches to the second set of equilibria it can not switch back.

To reiterate, the controller compensates for the energy loss due to damping by attempting to make the armature track a perfect cosine wave. This is accomplished through the use of feed forward, V_{eq}, and feedback to adjust for deviations from the desired trajectory, $\left[\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}\right]$. As the armature trajectory approaches a cosine wave the impact velocity tends toward zero (i.e. the velocity of the armature becomes a sin-wave with zero terminal velocity). Once the armature is brought near the catching coil, the controller switches to the second set of equilibria and stabilizes it in the desired final state.

A discontinuity is created in both the input, u, and the state, x, due to the switching from one equilibrium set to the other. This discontinuity does not effect the stability of the system since the controller renders both sets GAS. Let us consider the only two possibilities which can occur

1) The discontinuity occurs because the armature has passed the $z = 1$ mm boundary before $t = \pi/\omega$.
2) The discontinuity occurs at $t = \pi/\omega$ because the armature has not yet passed $z = 1$ mm.

In either case the discontinuity must occur and the controller must switch to the 2^{nd} set of equilibria defined by (43) (44) (45) (46). After the switch the system can be viewed as having initial conditions defined by the values of the states after the discontinuity. Since the controller can not switch back to the 1^{st} set of equilibria and the 2^{nd} set of equilibria is GAS, we conclude that the system is GAS despite the presence of the discontinuity.

V. Observer Design

The feedback designed in Sec. III is implemented using

1) An eddy current sensor to measure the position of the armature.
2) A current sensor to measure the current in the catching coil.
3) A nonlinear observer to estimate the armature velocity.

As described in Sec. II, an eddy current sensor mounted on the rear of the EVA detects the position of the armature by measuring changes in the magnetic field caused by the motion of the sensor target. The magnetic flux is determined through the relationship

$$\lambda = \frac{2k_a i}{k_b + z} \quad (48)$$

where i is the measured current in the magnetic coil.

The observer is then implemented as

$$\frac{d\hat{z}}{dt} = \hat{v} + \Gamma_1(z, \hat{z}) \quad (49)$$

$$\frac{d\hat{v}}{dt} = \frac{1}{m}\left(k_s(l - \hat{z}) - b\hat{v}\right) + \Gamma_2(\lambda, z, \hat{z}), \quad (50)$$

where

$$\Gamma_1(z, \hat{z}) = g_1(z - \hat{z}) \quad (51)$$

$$\Gamma_2(\lambda, z, \hat{z}) = -\frac{1}{m}\frac{\lambda^2}{4k_a} + g_2(z - \hat{z}). \quad (52)$$

Computing the error dynamics, $e = \begin{bmatrix} z - \hat{z} \\ v - \hat{v} \end{bmatrix}$,

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k_s}{m} & -\frac{b}{m} \end{bmatrix}}_{A_r}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_r}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}.$$

The gains g_1 and g_2 are selected based on a Kalman filter of (A_r, C_r) to achieve the desired performance.

VI. Experimental Results

Experimental results obtained using the nonlinear controller are given in Table I and Fig. 2. As seen the controller does indeed stabilize the armature against the magnetic coil while achieving a mean impact velocity of 0.12 m/s. The switch from the first equilibrium set to the second is seen at approximately 4.0 ms by the discontinuity in the input voltage. It is important to note that the real input to the system is a duty cycle command to the PWM drivers equal to the commanded voltage shown in Fig. 2 divided by the supply voltage of 180 V.

TABLE I

Statistical results for the GAS nonlinear controller.

	Impact Velocity
Mean	0.12 m/s
σ	0.07 m/s
Max	0.32 m/s
Min	0.04 m/s

In order to show stability experimentally the controller drives the system to a constant equilibrium corresponding to the armature hovering 2.0 mm away from the catching coil, Fig. 3. Thereby stabilizing the armature away from the physical constraints, i.e. the magnetic coils. As proven

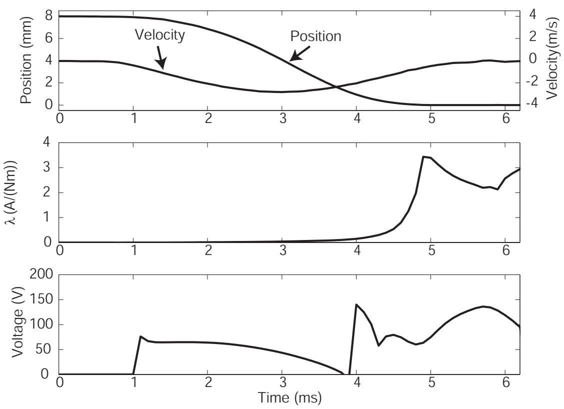

Fig. 2. Experimental results achieved using the GAS nonlinear controller.

in [15] all equilibrium points less than $\frac{2}{3}l$ mm (approximately 2.6 mm) away from the catching coil are open loop unstable. Hovering the armature often is not practical from a power consumption point of view, but may be advantageous under certain engine conditions or in other applications/systems that have similar dynamics to the EVA.

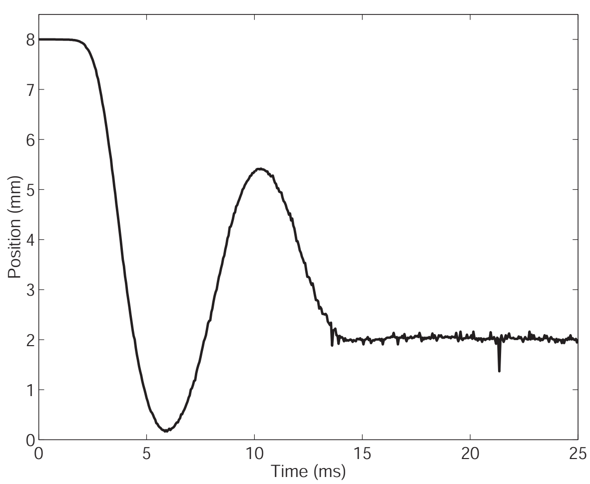

Fig. 3. Stabilizing the armature 2.0 mm away from the catching coil.

VII. Summary

This paper has presented a controller which renders the electromechanical valve actuator GAS without assuming linearity. Stability is both proven mathematically and shown experimentally. Future work will focus on improving the performance of the system through careful selection of the Lyapunov function as well as cycle to cycle based compensation [5], [10], [14]. In addition sensitivity to parameters and robustness issues will be investigated.

VIII. References

[1] S. Butzmann, J. Melbert, and A. Kock, "Sensor-less control of electromagnetic actuators for variable valve train," *SAE 2000-01-1225*.

[2] N. Cheung, K. Lim, and M. Rahman, "Modeling a linear and limited travel solenoid," *Proceedings of the IECON*, pp. 1567–1572, 1993.

[3] K. Hatano, K. Lida, H. Higashi, and S. Murata, "Development of a new multi-mode variable valve timing engine," *SAE 930878*.

[4] R. Henry and B. Lequesne, "A novel, fully flexible, electro-mechanical engine valve actuation system," *SAE 970249*.

[5] W. Hoffmann, K. Peterson, and A. Stefanopoulou, "Iterative learning control for soft landing of electromechanical valve actuator in camless engines," *IEEE Transactions on Control Systems Technology*, vol. 11, no. 2, pp. 174–184, March 2003.

[6] H. Khalil, *Nonlinear Systems*, 2nd ed. Prentice Hall, 1996.

[7] T. Leone, E. Christenson, and R. Stein, "Comparison of variable camshaft timing strategies at part load," *SAE 960584*.

[8] M. Levin and M. Schechter, "Camless engine," *SAE 960581*.

[9] M. Matsuki, K. Nakano, T. Amemiya, Y. Tanabe, D. Shimizu, and I. Ochmura, "Development of a lean burn engine with a variable valve timing mechanism," *SAE 960583*.

[10] K. Peterson, A. Stefanopoulou, Y. Wang, and M. Haghgooie, "Nonlinear self-tuning control for soft landing of an electromechanical valve actuator," *Proceedings of 2002 IFAC on Mechatronics*, pp. 207–212, Nov 2002.

[11] K. Peterson, A. Stefanopoulou, Y. Wang, and T. Megli, "Output observer based feedback for soft landing of electromechanical camless valvetrain actuator," *Proceedings of 2002 ACC*, pp. 1413–1418, May 2002.

[12] E. Sontag, "A universal construction of artstein's theorem on nonlinear stabilization," *Systems and Control Letters*, vol. 13, no. 2, pp. 117–123, 1989.

[13] R. Stein, K. Galietti, and T. Leone, "Dual equal vct- a variable camshaft timing strategy for improved fuel economy and emissions," *SAE 950975*.

[14] C. Tai, A. Stubbs, and T. Tsao, "Modeling and controller design of an electromagnetic engine valve," *Proceedings American Control Conference*, pp. 2890–2895, June 2001.

[15] C. Tai and T. Tsao, "Control of an electromechanical camless valve actuator," *Proceedings American Control Conference*, May 2002.

[16] Y. Wang, T. Megli, M. Haghgooie, K. Peterson, and A. Stefanopoulou, "Modeling and control of electromechanical valve actuator," *SAE 2002-01-1106*.

[17] C. Weddle and D. Leo, "Embedded actuation systems for camless engines," *Proceedings of Ninth International Conference on Adaptive Structures and Technologies*, October 1998.