The tables below refer to approximate values for the integral

$$\int_1^2 \frac{e^x}{x}\,dx\ .$$

The first table gives approximate values of this integral calculated by the right endpoint rule, left endpoint rule, trapezoidal rule, midpoint rule, and Simpson's rule. These approximations are tabulated for various values of n, the number of sub-intervals, where n is repeatedly doubled, that is, we take $n = 2^k$ for $k = 1, 2, \ldots, 8$. The second table gives the errors in these approximations.

Notice that for the left and right endpoint rules the error is approximately halved when n is doubled; that is, the error is roughly proportional to $1/n$. For the trapezoidal and midpoint rules the error decreases by a factor of 4 each time, that is, is roughly proportional to $1/n^2$, while for Simpson's rule the error the error decreases by a factor of 16, behaving as $1/n^4$. Note also that the error in the midpoint rule is roughly half that of the error in the trapezoid rule.

APPROXIMATIONS

n	Right	Left	Trapezoidal	Midpoint	Simpson
2	2.8530372710	3.3411603815	3.0970988263	3.0403093761	* * * * *
4	2.9466733236	3.1907348788	3.0687041012	3.0543352777	3.0592391928
8	3.0005043006	3.1225350783	3.0615196894	3.0579157666	3.0591248855
16	3.0292100336	3.0902254224	3.0597177280	3.0588159959	3.0591170742
32	3.0440130148	3.0745207092	3.0592668620	3.0590413816	3.0591165733
64	3.0515271982	3.0667810454	3.0591541218	3.0590977488	3.0591165417
128	3.0553124735	3.0629393971	3.0591259353	3.0591118418	3.0591165398
256	3.0572121577	3.0610256195	3.0591188886	3.0591153652	3.0591165396

ERRORS

n	Right	Left	Trapezoidal	Midpoint	Simpson
2	0.2060792686	-0.2820438419	-0.0379822866	0.0188071635	* * * *
4	0.1124432161	-0.1316183392	-0.0095875616	0.0047812620	-0.0001226532
8	0.0586122390	-0.0634185386	-0.0024031498	0.0012007731	-0.0000083459
16	0.0299065060	-0.0311088828	-0.0006011884	0.0003005437	-0.0000005346
32	0.0151035249	-0.0154041695	-0.0001503223	0.0000751580	-0.0000000336
64	0.0075893414	-0.0076645058	-0.0000375822	0.0000187909	-0.0000000021
128	0.0038040662	-0.0038228574	-0.0000093956	0.0000046978	-0.0000000001
256	0.0019043820	-0.0019090798	-0.0000023489	0.0000011745	0.0000000000