

HOMEWORK 9

Definition. Two sets A and B are *equinumerous*, written $A \approx B$, iff there exists a bijection $f : A \rightarrow B$.

Question 1. Suppose that A , B and C are any sets. Prove:

- (i) $A \approx A$.
- (ii) If $A \approx B$, then $B \approx A$.
- (iii) If $A \approx B$ and $B \approx C$, then $A \approx C$.

Question 2. Prove or disprove the following statements.

- (a) Suppose that A and B are any sets. If $A \approx B$ and $f : A \rightarrow B$ is an injection, then f is a bijection.
- (b) Suppose that A and B are any sets. If $A \approx B$ and $f : A \rightarrow B$ is a surjection, then f is a bijection.

Question 3. Let $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ be the function defined by

$$f(m, n) = 2^{m-1}(2n - 1).$$

- (a) Prove that f is an injection.
- (b) Prove that f is a surjection.