

Oral Qualifying Examination Syllabus

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1 Combinatorial Number Theory

- Bertrand's postulate; sum of reciprocals of primes; Mertens's theorem.
- Brun's pure sieve; upper bound for the number of twin primes not exceeding x ; convergence of the sum of reciprocals of twin primes.
- Schnirelmann density; connections with additive bases; primes as a basis.
- Squares as a basis; sums of three squares; Cauchy's polygonal number theorem.
- Roth's theorem; Van der Waerden's theorem; Schur's theorem; sum-free sets.
- Ordered and unordered partitions; Euler's pentagonal number theorem; Rogers-Ramanujan identities; elementary estimates for the number of unordered partitions.
- Erdős-Fuchs theorem; Sidon sets; B_h -sequences.

2 Diophantine Approximations

- Continued fractions; best approximations; Hurwitz's theorem; quadratic irrationals; Pell's equation.
- Lagrange and Markoff spectra; gaps in the Markoff spectrum; Hall's ray; Markoff's equation.
- Irrationality of e , π , $\ln 2$ and $\zeta(3)$; transcendence of e and π ; Lindemann-Weierstrass theorem; Gelfond-Schneider theorem.
- Approximability of algebraic numbers; Liouville's theorem and improvements.
- Distribution modulo 1; Weyl's criterion; uniform distribution of $\{n\alpha\}$ for irrational α ; uniform distribution of polynomials; normal numbers.

3 Combinatorial Game Theory

- Impartial games; standard and misère Nim; sums of games; Sprague-Grundy theorem.
- Surreal numbers; field structure and total ordering; positive, negative, zero and fuzzy games.
- Positional games; pairing strategy; Erdős-Selfridge theorem; potential functions.
- The digraph of a game; Richardson's theorem; uniqueness of kernel in acyclic digraphs.