

# Teaching Statement

For all the talk about absolute truth and the Platonic universe, mathematics is a human activity. We inherit morsels of wisdom from our teachers and pass them on to our students, hoping to recreate the moments of joy, awe and wonder that we ourselves experienced as students. It is the immense debt I owe to my teachers that persuaded me to consider a career in academia. I have found teaching mathematics a richly rewarding experience and an oasis of purpose in the middle of lean patches in my research career.

As the sole instructor for two upper level undergraduate courses – Numerical Analysis and Combinatorics – I was responsible for designing the syllabus, preparing lectures, conducting exams and awarding grades. In addition, I have been a teaching assistant for courses in differential and integral calculus since my second year of graduate study at Rutgers, working with 60-80 students each semester. I also had grading duties for three mathematics courses during my first year at Rutgers.

I taught Combinatorics during the summer of 2006 to a small class of undergraduate and graduate students. Although the number of lecture hours were the same as during a regular semester, the six-week term presented unique challenges. I am used to gentle introductions, but with a mid-term exam two weeks after the first lecture, an air of briskness hung over the classroom. And yet, the cramped schedule kept my students constantly on their toes, as they were only too aware that if they slacked off for two days, they've had it. They fared quite well in the end, and matched my expectations, as a class.

One could present combinatorics to students as a bouquet of pretty results, or one could outline the depth and structure that casual observers fail to see when they dismiss the subject as “puzzles and such”. I took a convenient middle ground, starting with mathematical induction and the pigeonhole principle, spending two weeks on counting problems, recurrences, generating functions, partitions, and the like, giving an overview of error-correcting codes and Latin squares, taking the class on a quick tour of graph theory, and rounding things off with the probabilistic method, Ramsey theory and combinatorial games – emphasising connections all the while. I taught them nearly everything I knew as an undergraduate, and a few topics more.

I learned a lot too. I realised the value of good homework problems and the effectiveness of strategically placed hints. I found that it is worthwhile to think about possible homework and exam problems from a topic before presenting it in the lecture, so that the examples demonstrated in class are sufficiently empowering. I discovered that a couple of cute exercises can give an entire homework set the appearance of a challenge rather than a chore. I remember a problem where it was required to develop a strategy to guess a number from 0 to 15 using seven Yes/No questions, the respondent being allowed to lie once. It was clear from the writeups that almost every student in the class had given it considerable thought. The cognoscenti reduced it to Hamming codes, but the more memorable answers consisted of search trees with questions like “Is the number less than 8 and prime?”

I also had the opportunity to teach Numerical Analysis during the summer of 2005. We looked at rates of convergence and error estimates for a variety of root-finding, interpolation and quadrature schemes. Error analysis was invariably preceded by some variant of “the worst that could happen”. Proofs were presented, but seldom advertised as such. We also discussed numerical methods for solving ordinary differential equations. Programming projects were designed as a supplement to regular homework, so that students get a feel of how the methods work in practice.

As a researcher, I work on problems that I think are beautiful, and am rather indifferent to real-life applications. As an instructor, however, I try to put theorems and techniques in context, and emphasise connections between different areas of mathematics and also consequences in other disciplines. My students have consistently taken a favourable view of my enthusiasm, quizzing me on details even when I go off on tangents. In fact, such excursions have often proved more popular than the regular textbook fare, and I have smiled wistfully when the bolder students demand to know why, for example, discrete mathematics is not part of their core curriculum. I have wondered about this myself. For all the utility and applicability of freshman calculus, its contribution towards developing mathematical maturity has appeared rather negligible to me.

I believe that mathematical reasoning is taught best through accessible proofs. While students may find this intimidating at first, it is only a matter of time before they rise to the occasion and chip in with “suppose not” and “now we are done by induction”. Underestimating the abilities of students is tantamount to doing them a disservice. The apparent incompetence of a student can often be remedied by purging muddled concepts and reinforcing the fundamentals in about half an hour: for example, it helps to clarify right at the beginning of a first course in calculus that most functions are not linear, that inequalities sometimes get reversed, and that the converse of a theorem need not always hold.

Although I adhere to conventional methods of lecturing and presenting material, I strongly believe in the use of technology to aid teaching. As an independent instructor and as a teaching assistant, I make it a point to post notes and study material on dedicated webpages. I also encourage my students to clarify doubts via email. At Rutgers, we use WeBWorK (developed at the University of Rochester) as part of our evaluation scheme for the freshman differential calculus course. Accounts are set up online to allow students repeated attempts at homework problems, with a record of all previous attempts viewable by instructors. Wrong answers often tell us a lot about the difficulties a student might be experiencing with the material. Such individual feedback is valuable and would normally be impossible to obtain.

I do have reservations on the extent to which calculators are deemed necessary in the classroom. It is often argued that mathematical ability has nothing to do with numerical proficiency, and that calculators shield students from the drudgery of hand-computation.

I disagree: in my experience, students constrained to perform approximate computations using pen and paper pick up Taylor series in a jiffy, while button-pushers remain blissfully ignorant of Taylor and his series. My candidate for the debilitating malady of the new millennium is *innumeracy*.

Whenever possible, I try to make learning interactive. During one of my combinatorics lectures, I asked each student in the class to write down the birthdays of two people they know and predicted a clash. This was not very safe, since the class was quite small and the probability of a clash was only about 0.6. But events did unfold as I had hoped, and my students were astonished. What they saw in action was, of course, the phenomenon known in probability theory as the “birthday paradox”, which ceases to be a mystery once it is clarified that it is not so much the size of the group as the number of pairs in the group that matters.

I believe in reaching out to students individually. Many a time I grind to an abrupt halt in the middle of a computation, call a student by name and ask for input. If the response is wrong, I demonstrate the gap between what was suggested and what is sought. The correct answer usually follows immediately, often from the same source as before. Teaching, as Richard Bach aptly observed, is “reminding others that they know just as well as you.”