MATHEMATICS 300 — FALL 2017

Introduction to Mathematical Reasoning

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HOMEWORK ASSIGNMENT NO. 10, DUE ON WEDNESDAY, NOVEMBER 15

I. Problems 42, 43 and 44 from the ninth set of lecture notes.

II. If $a \in \mathbb{R}$, $\varepsilon \in \mathbb{R}$, and $\varepsilon > 0$, then the <u>ε-neighborhood</u> of a is the set $N_\varepsilon(a)$ given by

$$N_\varepsilon(a) = \{\, x \in \mathbb{R} : a - \varepsilon < x < a + \varepsilon \,\}\,.$$

A subset U of $\mathbb{R}$ is <u>open</u> if the following is true:

(*) for every member a of U there exists a positive real number ε such that $N_\varepsilon(a) \subseteq U$.

Prove that

1. If U and V are open subsets of $\mathbb{R}$, then $U \cap V$ and $U \cup V$ are open.
2. If a, b are real numbers, then the set $]a, b[$ given by

$$]a, b[= \{x \in \mathbb{R} : a < x \wedge x < b\}$$

is an open set.

3. If a, b are real numbers and $a \le b$, then the set $[a, b]$ given by

$$[a, b] = \{x \in \mathbb{R} : a \le x \wedge x \le b\}$$

is not an open set.

III. If a, b, c are integers, a <u>greatest common divisor</u> of a, b and c is an integer g such that

1. g divides a, g divides b, and g divides c.

2. If d is an arbitrary integer such that d divides a, d divides b, and d divides c, it follows that $d \leq g$.

A. ***Prove*** that if $a \in \mathbb{Z}$, $b \in \mathbb{Z}$, $c \in \mathbb{Z}$, and $a \neq 0 \vee b \neq 0 \vee c \neq 0$ then

   1. there exists a unique integer g such that g is a geatest common divisor of a, b, and c,
   2. the integer g satisfies:
      (a) $g \in \mathbb{N}$,
      (b) there exist integers u, v, w such that $g = ua + vb + wc$.

B. ***Prove or disprove*** each of the following results:

   1. If a, b, c are integers none of which is equal to zero, and the greatest common divisor of a, b, c is equal to 1, and n is an integer that is divisible by a, b and c, then n is divisible by the product abc.
   2. If a, b, c are integers none of which is equal to zero, and the greatest common divisors $GCD(a, b)$, $GCD(a, c)$, $GCD(b, c)$ are equal to 1, and n is an integer that is divisible by a, b and c, then n is divisible by the product abc.