

# MATHEMATICS 300 — FALL 2018

## *Introduction to Mathematical Reasoning*

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### **HOMEWORK ASSIGNMENT NO. 2, DUE ON TUESDAY, SEPTEMBER 25**

#### **Problem 1.**

- (i) Book, Page 41, Problem 11, parts (b), (d), (e).
- (ii) Book, Page 50, Problem 12, parts (a),(c), (d), (e), (f).

*Please read carefully the answers to the starred questions given at the end of the book, in order to have an idea of what you are expected to do. I would advice you to be harsher than the book is. For example: Proof (a) of Problem 11 is wrong because just the very first line is not right, because “Let  $a$ ” is not a sentence, and that’s enough to invalidate the whole proof. (In the book the authors try to be kinder, and say that “let  $a$ ” is an incomplete sentence.)*

**Problem 2.** Book, Page 49, Problem 4, parts (a), (c), (d), (e). Feel free to do proofs by contradiction instead of proofs by contraposition as the book asks you to do. (See the remark on the next page.)

**Problem 3.** Book, Page 49, Problem 5.

**Problem 4.** Book, Page 50, Problem 11.

**Problem 5.** Book, Page 61, Problem 3.

**Problem 6.** Book, Page 61, Problem 5, part (b).

**REMARK:** I personally do not believe there is a need to even consider “proofs by contraposition”. **REASON:** If you can prove  $P \implies Q$  by contraposition (that is, by proving  $\sim Q \implies \sim P$  instead), then you can also prove it by contradiction:

Assume  $P$ .

We want to prove  $Q$ .

We will prove  $Q$  by contradiction.

Assume  $\sim Q$ .

Insert here your proof of  $\sim Q \implies \sim P$ .

$\sim Q \implies \sim P$ .

Since we have  $\sim Q$  and  $\sim Q \implies \sim P$ , it follows by Modus Ponens that

$\sim P$ .

So  $P \wedge \sim P$ , which is a contradiction.

Since we have proved a contradiction assuming  $\sim Q$ , we have proved

$Q$ .

Since we have proved  $Q$  assuming  $P$ , we get

$P \implies Q$ .

**Q.E.D.**