MATHEMATICS 300 — FALL 2018

Introduction to Mathematical Reasoning

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HOMEWORK ASSIGNMENT NO. 9, DUE ON TUESDAY, NOVEMBER 27.

PROBLEM 1. Problem 27 from the notes, Part IV, page 66.

PROBLEM 2. Problem 30 from the notes, Part IV, page 69

PROBLEM 3. Using the inductive definition of "power" of a real number, given on page 75 of the notes, Part IV, ***prove*** that

1. If $a \in \mathbb{R}$ and m, n are natural numbers, then $a^m a^n = a^{m+n}$.
2. If $a \in \mathbb{R}$ and m, n are natural numbers, then $\Big(a^m\Big)^n = a^{mn}$.

PROBLEM 4. Using the inductive definition of summation given on page 77 of the notes, Part IV, ***prove*** that

(*) If $n \in \mathbb{N}$ and $(a_1, a_2, \ldots, a_n)$, $(b_1, b_2, \ldots, b_n)$ are two lists of n real numbers, then

$$\Big(\sum_{j=1}^{n} a_j\Big) + \Big(\sum_{j=1}^{n} b_j\Big) = \sum_{j=1}^{n}(a_j + b_j)\,.$$

(NOTE: For $n = 2$, this says that

$$(a_1 + a_2) + (b_1 + b_2) = (a_1 + b_1) + (a_2 + b_2)$$

which is a simple ocnsequence of the commutative and associative laws of additon, as follows:

$$\begin{aligned}
(a_1 + a_2) + (b_1 + b_2) &= ((a_1 + a_2) + b_1) + b_2 \\
&= (a_1 + (a_2 + b_1)) + b_2 \\
&= (a_1 + (b_1 + a_2)) + b_2 \\
&= ((a_1 + b_1) + a_2) + b_2 \\
&= (a_1 + b_1) + (a_2 + b_2)\,.
\end{aligned}$$

What I am asking you to do is extend the same result to sums of any length n. HINT: Do induction with respect to n. Here is the inductive step to go from 2 to 3, that is, the implication $P(2) \Longrightarrow P(3)$:

$$\begin{aligned}
(\sum_{j=1}^{3} a_j) + (\sum_{j=1}^{3} b_j) &= (\sum_{j=1}^{3} a_j) + \big((\sum_{j=1}^{2} b_j) + b_3\big) \\
&= \big((\sum_{j=1}^{2} a_j) + a_3\big) + \big((\sum_{j=1}^{2} b_j) + b_3\big) \\
&= \Big(\big((\sum_{j=1}^{2} a_j) + a_3\big) + (\sum_{j=1}^{2} b_j)\Big) + b_3 \\
&= \Big((\sum_{j=1}^{2} a_j) + \big((a_3 + (\sum_{j=1}^{2} b_j)\big)\Big) + b_3 \\
&= \Big((\sum_{j=1}^{2} a_j) + \big((\sum_{j=1}^{2} b_j) + a_3\big)\Big) + b_3 \\
&= \Big(\big((\sum_{j=1}^{2} a_j) + (\sum_{j=1}^{2} b_j)\big) + a_3\Big) + b_3 \\
&= \Big(\big(\sum_{j=1}^{2}(a_j + b_j)\big) + a_3\Big) + b_3 \\
&= \big(\sum_{j=1}^{2}(a_j + b_j)\big) + (a_3 + b_3) \\
&= \sum_{j=1}^{3}(a_j + b_j)\,.
\end{aligned}$$