MATHEMATICS 300 — FALL 2019

Introduction to Mathematical Reasoning

H. J. Sussmann

HOMEWORK ASSIGNMENT NO. 3, DUE ON MONDAY, SEPTEMBER 30

For each of the following sentences in formal language

i. ***Indicate*** which of the variables that occur in the sentence are open (i.e., free) and which ones are closed (i.e., bound, or dummy).

ii. ***Indicate*** whether the sentence is a proposition (i.e., has no free variables) or not.

iii. If the sentence is a proposition, ***indicate*** whether it is true or false, and ***prove*** it, if it is true, or ***disprove*** it, if it is false.

iv. If the sentence is not a proposition, ***give examples*** of values of the free variables for which the sentence is true, or ***prove*** that no such examples exist, and ***give examples*** of values of the free variables for which the sentence is false, or ***prove*** that no such examples exist.

1. $ax = b.$ (a, x, b real numbers)
2. $(\exists x \in \mathbb{R})ax = b$ (a, x, b real numbers)
3. $(\forall b \in \mathbb{R})(\exists x \in \mathbb{R})ax = b.$ (a, x, b real numbers)
4. $(\forall a \in \mathbb{R})(\exists x \in \mathbb{R})ax = b.$ (a, x, b real numbers)
5. $(\forall a \in \mathbb{R})(\forall b \in \mathbb{R})(\exists x \in \mathbb{R})ax = b.$ (a, x, b real numbers)
6. $(\forall a \in \mathbb{R})(\exists b \in \mathbb{R})(\exists x \in \mathbb{R})ax = b.$ (a, x, b real numbers)
7. $(\forall b \in \mathbb{R})(\exists a \in \mathbb{R})(\exists x \in \mathbb{R})ax = b.$ (a, x, b real numbers)
8. $(\exists a \in \mathbb{R})(\exists b \in \mathbb{R})(\forall x \in \mathbb{R})ax = b.$ (a, x, b real numbers)
9. $(\forall m \in \mathbb{N})(\exists n \in \mathbb{N})n > m.$ (m, n natural numbers)
10. $(\forall m \in \mathbb{N})(\exists n \in \mathbb{N})n < m.$ (m, n natural numbers)
11. $(\forall m \in \mathbb{Z})(\exists n \in \mathbb{Z})n < m.$ (m, n integers)
12. $(\exists n \in \mathbb{Z})(\forall m \in \mathbb{Z})n < m.$ (m, n integers)
13. $(\exists n \in \mathbb{N})(\forall m \in \mathbb{N})n < m.$ (m, n natural numbers)

14. $(\exists n \in \mathbb{N})(\forall m \in \mathbb{N}) n \leq m.$ (m, n natural numbers)
15. $(\exists m \in \mathbb{Z})(\exists n \in \mathbb{Z}) m^2 - n^2 = 28.$ (m, n integers)
16. $(\exists m \in \mathbb{Z})(\exists n \in \mathbb{Z}) m^2 - n^2 = 29.$ (m, n integers)
17. $(\exists m \in \mathbb{Z})(\exists n \in \mathbb{Z}) m^2 - n^2 = 30.$ (m, n integers)
18. $(\exists m \in \mathbb{Z})(\exists n \in \mathbb{Z}) m^2 - n^2 = k.$ (m, n, k integers)
19. $(\forall k \in \mathbb{Z})(\exists m \in \mathbb{Z})(\exists n \in \mathbb{Z}) m^2 - n^2 = k.$ (m, n, k integers)
20. $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R}) x + z = y.$ (x, y, z real numbers)
21. $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R}) x + z^2 = y.$ (x, y, z real numbers)
22. $(\exists z \in \mathbb{R}) x \cdot z = y.$ (x, y, z real numbers)
23. $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R}) x \cdot z = y.$ (x, y, z real numbers)
24. $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R}) (1 + x^2) \cdot z = y.$ (x, y, z real numbers)
25. $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{R}) (1 - x^2) \cdot z = y.$ (x, y, z real numbers)