# MATHEMATICS 300 — FALL 2019

*Introduction to Mathematical Reasoning*

*H. J. Sussmann*

## HOMEWORK ASSIGNMENT NO. 4, DUE ON MONDAY, OCTOBER 7

For each of the following sentences in formal language

i. ***Translate*** the sentence into English. Please use a translation that is a reasonable English sentence. For example:
   - do ***not*** translate "$n \in \mathbb{N}$" as "$n$ belongs to the set of natural numbers"; write "$n$ is a natural number", because that is a much better translation;
   - do ***not*** translate "$2|n$" as "2 divides $n$", or "$n$ is divisible by 2"; write "$n$ is even", because that is a much better translation;
   - do ***not*** translate "$(\forall n \in \mathbb{N})(2|n \Longrightarrow \sim 2|n+1)$" as "for all $n$ in the set of natural numbers if $n$ is divisible by 2 then it's not true that $n+1$ is divisible by 2; write "if $n$ is an even natural number then $n+1$ is not even".

ii. ***Indicate*** which of the variables that occur in the sentence are open (i.e., free) and which ones are closed (i.e., bound, or dummy).

iii. ***Indicate*** whether the sentence is a proposition (i.e., has no free variables) or not.

iv. If the sentence is a proposition, ***indicate*** whether it is true or false, and ***prove*** it, if it is true, or ***disprove*** it, if it is false.

v. If the sentence is not a proposition, then
   - ***give examples*** of values of the free variables for which the sentence is true, and ***explain why*** the sentence is true for those values, or ***prove*** that no such examples exist,

   and

   - ***give examples*** of values of the free variables for which the sentence is false, and ***explain why*** the sentence is true for those values, or ***prove*** that no such examples exist.

1. $ax = bx \Longrightarrow a = b.$ $(a, x, b$ real numbers$)$
2. $(\exists x \in \mathbb{R})ax = bx \Longrightarrow a = b.$ $(a, x, b$ real numbers$)$

3. $(\exists x \in \mathbb{R})(ax = bx \Longrightarrow a = b)$. ($a, x, b$ real numbers)
4. $(\forall x \in \mathbb{R})ax = bx \Longrightarrow a = b$. ($a, x, b$ real numbers)
5. $(\forall x \in \mathbb{R})(ax = bx \Longrightarrow a = b)$. ($a, x, b$ real numbers)
6. $(\forall n \in \mathbb{N})(\exists M \in \mathbb{N})(\forall m \in \mathbb{N})\Big(m > M \Longrightarrow \frac{1}{m} < \frac{1}{3n}\Big)$.
   ($m, M, n$ natural numbers)
7. $(\forall x \in \mathbb{R})\Big(x > 0 \Longrightarrow x^2 > x$. ($x$ a real number)
8. $(\forall n \in \mathbb{N})(\exists m \in \mathbb{N})2m < n$. ($m, n$ natural numbers)
9. $(\forall n \in \mathbb{N})(\exists m \in \mathbb{N})2m > n$. ($m, n$ natural numbers)
10. $(\exists m \in \mathbb{N})(\forall n \in \mathbb{N})2m > n$. ($m, n$ natural numbers)
11. $(\exists x \in \mathbb{R})(x^2 - 2x < 0 \wedge x^2 - 6x - 8 < 0)$. ($x$ a real number)
12. $(\forall n \in \mathbb{N})\Big((2|n \wedge n > 2) \Longrightarrow 2^n - 1 \text{ is not prime}\Big)$.
    ($n$ a natural number)
13. $(\forall n \in \mathbb{Z})\Big((a|n \wedge b|n) \Longrightarrow ab|n\Big)$. ($n, a, b$ integers)
14. $(\forall a \in \mathbb{Z})(\forall b \in \mathbb{Z})(\forall n \in \mathbb{Z})\Big((a|n \wedge b|n) \Longrightarrow ab|n\Big)$. ($n, a, b$ integers)