MATHEMATICS 361 — FALL 2019
SET THEORY

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HOMEWORK ASSIGNMENT NO. 1, DUE ON THURSDAY, SEPTEMBER 12

1. Book, Pages 6,7, Exercises 1,2 3, 4.

2. Book, Pages 9, Exercises 5,7.

3. *This problem is about "pure set theory"; that is, we assume there are no atoms, so everything is a set. Hence "$\forall x$" means "for every set x", and "$\exists x$" means "there exists a set x such that".*

   A set a is <u>transitive</u> if every member of A is a subset of A. (That is, a is <u>transitive</u> if the following holds: $(\forall u)(u \in a \Longrightarrow u \subseteq a)$.)

   Prove that

   i. the empty set is transitive;

   ii. the union of two transitive sets is transitive, that is:

   $$\forall a \forall b \Big[\Big(\forall u (u \in a \Longrightarrow u \subseteq a) \ \&\ \forall u (u \in b \Longrightarrow u \subseteq b) \Big) \Longrightarrow \forall u (u \in a \cup b \Longrightarrow u \subseteq a \cup b) \Big] \, ;$$

   iii. if a is a transitive set, then the set $a \cup \{a\}$ is transitive;

   iv. the power set $\mathfrak{P}a$ of a transitive set a is transitive. (That is:

   $$\forall a \Big[\forall u (u \in a \Longrightarrow u \subseteq a) \quad \Longrightarrow \quad \forall u (u \in \mathfrak{P}a \Longrightarrow u \subseteq \mathfrak{P}a) \Big] \, .)$$