

MATHEMATICS 300 — FALL 2009

Introduction to Mathematical Reasoning

H. J. Sussmann

INSTRUCTOR'S NOTES

(September 3, 2009)

1 Information on the course

1.1 About the instructor

My *name* is **H. J. Sussmann**. My *office* is **Hill 538**.

My *Rutgers phone number* is 732-445-2390, extension 5407.

My *e-mail address* is **sussmann@math.rutgers.edu**.

1.2 Web page

I have set up a *Web page* for our Math 300 section:

<http://www.math.rutgers.edu/~sussmann/math300page-Fall09.html>

All the instructor's notes will be available there.

1.3 The lectures

We will have **26 lectures**, on

- September 1, 3, 10, 15, 17, 22, 24, 29
- October 1, 6, 8, 15, 20, 22, 27, 29
- November 3, 5, 10, 12, 17, 19, 24
- December 1, 8, 10

1.4 The midterms

There will be **two midterm exams**, on **Tuesday, October 13** and **Thursday December 3**.

1.5 Your final grade

- *Homework* will count for about 30% of your grade.
- The *two midterms* will count—together—for about 35%.
- The *final exam* will count for the remaining 35%.

1.6 Office hours

My office is **Hill 538**. My office hours will be:

- **Monday and Wednesday, 2:30pm to 4:30pm**, in my office,
- any other time (possibly including weekends), by appointment, in my office.

1.7 Textbook and notes

We will be using:

- the book *A Transition to Advanced Mathematics* (sixth edition), by Douglas Smith, Maurice Eggen, and Richard St. Andre;
- the notes written by the instructor.

The material of the instructor's notes is an integral part of the course, as much as that of the book. *Furthermore, the notes contain all kinds of important information. For example, in this set of notes there are lots of things you need to know in order to do your homework.*

1.8 Always bring the book to class!

In the lectures, we are going to spend a lot of time looking at the book and analyzing definitions, arguments and proofs given there. So

**Please always bring the book to class!
You are going to need it.**

1.9 Readings for the first 3 weeks (September 1, 3, 10, 15, and 17)

- the book's "Preface to the student,"
- the book's Chapter 1 (**all of it!**),
- the instructor's notes handed out during the first three weeks.

1.10 Homework assignment no. 1, due on Thursday September 10

Before you start writing your homework, read carefully the rest of this handout, in particular §1.11 on “some remarks about mathematical writing.”

1. Book, Exercises 1.1 (pages 8-9-10-11): Problems 5 (non-starred items), 8 (non-starred items), and 13 (non-starred items),
2. Book, Exercises 1.2 (pages 17-18-19-20): Problems 5 (non-starred items), 8 (non-starred items), and 13 (non-starred items),
3. (i) Prove or disprove: there exist integers x, y such that $x^2 - y^2 = 28$.
 (ii) Prove or disprove: there exist integers x, y such that $x^2 - y^2 = 29$.
 (iii) Prove or disprove: there exist integers x, y such that $x^2 - y^2 = 30$.

NOTE: to prove that there exists an object x such that a statement $S(x)$ involving x is true, you can exhibit one. (This is called Rule \exists_{prove} , for reasons that will be made clear below.) For example: to prove that there exists an integer x such that $x^2 + 1 = 10$, you can just say:

Let $x = 3$.

Then x is an integer.

And $x^2 + 1 = 10$.

So there exists an integer x such that $x^2 + 1 = 10$.

To disprove the assertion that there exists an object x such that a statement $S(x)$ involving x is true (i.e., to prove that there does not exist x such that $S(x)$ is true), you can do it by showing that for every possible x the sentence $S(x)$ is not true. For example, to prove that there does not exist a real number x such that $x^2 + 1 = 0$, you can just say:

Let x be a real number.

Then $x^2 \geq 0$.

So $x^2 + 1 > 0$.

Hence $\sim x^2 + 1 = 0$.

Therefore there does not exist a real number x such that $x^2 + 1 = 0$.

4. Prove that every year must have a Friday the 13th.

1.11 Some remarks about mathematical writing

1.11.1 Write clearly in complete sentences

You should write so that you can be easily understood by a properly trained English-speaking individual. In particular, this means that you must

- Use *complete English sentences*, that make clearly identifiable *statements* with a *clear meaning* that can be understood by anyone reading what you wrote. For example:

- If you tell me that “she is very smart,” but you haven’t told me who “she” is, then I don’t know who you are talking about, so you haven’t made a statement with a clear meaning.
 - If you write “ $x > 0$,” but you haven’t told me who “ x ” is, then I don’t know what you are talking about, so you haven’t made a statement.
 - If I ask you to state Pythagoras’ theorem and your answer only says “ $a^2 + b^2 = c^2$,” then nobody will know what you are talking about¹, because you have not said what “ a ,” “ b ,” and “ c ” are supposed to be.²
- Avoid exaggerated or incorrect use of cryptic mathematical notation.
 - Explain what you are doing.
 - Make sure that letter “variables” are used correctly, that is that either: (i) it has been said before what these letters stand for, or (ii) they are “closed variables” (or “dummy variables,” or “bound variables”) in the sense that will be discussed in detail in class, and will also be explained later in these notes.
 - Provide proper connectives between equations as well as between ideas.
 - Make sure that all the rules of English grammar (including those of spelling and punctuation) are strictly obeyed. (Here are two very entertaining books about punctuation that I recommend to you: (1) *Eats, Shoots and Leaves; the Zero Tolerance Approach to Punctuation*, by Lynne Truss, (2) *Eats, Shoots and Leaves; why Commas Really Do Make a Difference!*, by Lynne Truss and Bonnie Timmons.)
 - Try to say things correctly, following all the rules, but *in your own words*. Please *no rote learning*. If you have to memorize a definition or a statement, then that is not a good sign, because it indicates lack of understanding.
 - Please proofread carefully what you hand in. Ideally, you should read and reread and revise almost any formal communication. **Neatness and clarity count**, as you well know if you’ve tried to read any complicated document.
 - *Do not assume that the people reading your paper can read your mind. Do assume that they are intelligent, but also assume that they are busy, and cannot and will not spend an excessive amount of time puzzling*

¹Of course, your teacher will know what you are trying to say, and anybody who already knows the statement of Pythagoras’ theorem will know. But when you are asked to state a theorem or a definition **you should write it as if you were talking to somebody who does not know yet what the theorem or the definition say.**

²Here is a correct statement of Pythagoras’ theorem: *Let c be the length of the hypotenuse of a right triangle, and let a , b be the lengths of the other two sides. Then $a^2 + b^2 = c^2$.*

out your meaning. Communication is difficult, and written technical communication is close to an art.

Effective written exposition will be worth at least 50% of your grade. Conversely, bad or unclear exposition may be penalized as much as 50% of the grade or even more.

- The best reference known to me on effective writing is *The Elements of Style* by Strunk and White, a very thin paperback published by Macmillan. It isn't expensive, and it is easy to read. I recommend it.

1.11.2 Your written work

You should pay attention to presentation, especially for the homework:

- A nicely typed homework (e.g., using a word processor) is preferable to handwritten work. Handwritten work is acceptable too, but in that case:
 - If you have to cross out lots of words, then you should rewrite the whole thing anew, **cleanly and neatly**. If you are not willing to spend some of your time doing this; if what you hand in shows that you were in a hurry and that you did not make the effort to write things neatly and properly, then there is no reason for the instructor or the grader to spend any of our time reading what you wrote, and we will not do it.
 - Use a pen. Never use a pencil.
 - Use any color other than red (for example, black, blue, or green), but **DO NOT USE RED**. (Reason: The use of red is reserved for the instructor's and grader's comments.)
 - If you tear off the sheets from a spiral notebook, please make sure before you hand them in that there are none of those ugly hanging shreds of paper at the margins. Use scissors, or a cutter, if necessary.
- Make sure that **your name appears in every sheet of paper you hand in**, and that if you are handing in more than one sheet then the sheets are **stapled** and **numbered**.

If you hand in a homework assignment that has one of the following flaws:

- it is written carelessly or in a hurry,
- it has lots of words crossed out,
- it has unreadable handwriting,
- it has unstapled sheets,
- it has unnumbered sheets,
- it has sheets that fail to show your name,
- it has shreds of paper at the margins,
- it is written using pencil rather than a pen,
- it is written in red,

then you will lose points. If it has two or more of those flaws, then the assignment will be marked “unacceptable” and returned unread and, from Assignment No. 3 on, you will not get a chance to redo it and hand it in again.