

MATHEMATICS 300 — FALL 2009

Introduction to Mathematical Reasoning

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INSTRUCTOR'S NOTES

(October 22, 2009)

1 Homework assignment No. 8, due on Thursday October 29.

1. Let

$$A = \{x \in \mathbb{R} : x^3 - 6x + 5 = 0\},$$

and let

$$B = \left\{1, -\frac{1}{2} + \frac{\sqrt{21}}{2}, -\frac{1}{2} - \frac{\sqrt{21}}{2}\right\}.$$

Prove that $A = B$.

2. Prove that

$$(\forall A)(\forall B)(A = B \iff (A \subseteq B \wedge B \subseteq A)).$$

(Remember that “ $X \subseteq Y$ ” is read as “ X is a subset of Y ”, and means “ $(\forall x)(x \in A \implies x \in B)$ ”. Remember also that *capital letter variables* stand for sets, so that, for example, “ $(\forall A)$ ” means “for all sets A ”, and “ $(\exists A)$ ” means “there exists a set A such that”.)

3. Let

$$A = \{1, 3, 5, 6\},$$

let

$$B = \left\{n \in \mathbb{N} : (\exists k)(k \in A \wedge n = k^2 + k + 41)\right\},$$

and let

$$C = \{n \in \mathbb{N} : n \text{ is prime}\}.$$

Prove that $B \subseteq C$.