MATHEMATICS 300 — FALL 2009

Introduction to Mathematical Reasoning

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INSTRUCTOR'S NOTES

(December 6, 2009)

1 Homework assignment No. 13, due on Thursday, December 10, 2009

1. Book, Page 187, Problems 11, 12, and 16.

2. If A, B, C are sets, the *composite* of a function $f : A \mapsto B$ and a function $g : B \mapsto C$ is the function $g \circ f : A \mapsto C$ given by

$$(g \circ f)(x) = g(f(x)) \qquad \text{for} \quad x \in A\,.$$

Prove that composition of functions is associative, that is: If A, B, C, D are sets, and $f : A \mapsto B$, $g : B \mapsto C$, $h : C \mapsto D$, are functions, then $h \circ (g \circ f) = (h \circ g) \circ f$.

3. Suppose that A, B, C are sets, and $f : A \mapsto B$, $g : B \mapsto C$ are functions. Prove or disprove each of the following:

(i) If f and g are one-to-one then $g \circ f$ is one-to-one.

(ii) If $g \circ f$ is one-to-one then f is one-to-one.

(iii) If $g \circ f$ is one-to-one then g is one-to-one.

4. We know from an earlier problem that $\emptyset : \emptyset \mapsto \emptyset$. Prove that this function is one-to-one.