MATHEMATICS 432 — SPRING 2016
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HOMEWORK ASSIGNMENT NO. 1,
DUE ON THURSDAY, JANUARY 29
HAND IN PROBLEMS 3, 4, 6, 7, 9, 10


2. Book, Exercise 1.1.5 on Page 7.


4. A conic section is a curve obtained by intersecting a conic surface with a plane. *(You should read the article entitled “conic section” in Wikipedia. And, in particular, you should look at the pictures.)* Let $S$ be the surface in $\mathbb{R}^3$ consisting of all the points $(x, y, z)$ that satisfy $x^2 + y^2 = z^2$, so $S$ is a conic surface. Let $P$ be the plane in $\mathbb{R}^3$ with equation $z = ax + b$, where $a$ and $b$ are real numbers such that $0 < a < 1$ and $b > 0$. Show that $S \cap P$ is an ellipse.


6. Book, Exercise 1.2.4 on Page 12. *(NOTE: This problem is very important.)* The fact that “the straight-line segment is the shortest distance between two points” is one of the oldest and most useful mathematical results. A large part of our course will be devoted to the study of curved surfaces, and one of the main things we will study for a curved surface $S$ is precisely which curves in $S$ have this property of being “the shortest path between two points”. These curves are called “geodesics”, and understanding them is not at all a trivial task. Take, for example, the surface of the Earth. What is the shortest path from New York City to Beijing? Since both cities are approximately at the same latitude ($40^\circ$ North), you may think that the shortest way to fly from New York to Beijing is to follow the $40^\circ$ parallel, i.e., to fly due West from New York until you get to Beijing. But this is not at all the case! The parallel is not a geodesic. Planes fly from New York to Beijing follow a polar route, because it is much shorter. Describing the
geodesics on a curved space is difficult and important. We will have a lot to say about this question later.