

# MATHEMATICS 300 — SPRING 2015

## *Introduction to Mathematical Reasoning*

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### **HOMEWORK ASSIGNMENT NO. 1, DUE ON TUESDAY, JANUARY 27**

1. Translate each of the following formulas into a statement in ordinary English, indicate whether the statement is true or false, and explain why. Make your translation as English-sounding as possible. (For example, do not say “ $n$  is a member of the set of integers”. Say “ $n$  is an integer.”):
  - i.  $(\forall n \in \mathbb{N})(\exists m \in \mathbb{N})m > n$ .
  - ii.  $(\forall n \in \mathbb{N})(\exists m \in \mathbb{N})m < n$ .
  - iii.  $(\forall n \in \mathbb{N})(\exists m \in \mathbb{N})m \leq n$ .
  - iv.  $(\forall n \in \mathbb{Z})(\exists m \in \mathbb{Z})m > n$ .
  - v.  $(\forall n \in \mathbb{Z})(\exists m \in \mathbb{Z})m < n$ .
  - vi.  $(\forall n \in \mathbb{Z})(\exists m \in \mathbb{Z})m \leq n$ .
  - vii.  $(\exists m \in \mathbb{Z})(\forall n \in \mathbb{Z})m < n$ .
  - viii.  $(\forall x \in \mathbb{R})(x > 0 \implies (\exists y \in \mathbb{R})(y > 0 \wedge y < x))$ .
  - ix.  $(\forall x \in \mathbb{R})(x > 0 \wedge (\exists y \in \mathbb{R})(y > 0 \wedge y < x))$ .
2. Write the statement of Theorem 2 of Lecture 2 in formal language, as is done in the Lecture 2 notes for Theorem 1.
3. Prove Theorem 2 of the notes for Lecture 2.
4. Write the statement of Theorem 4 of the notes for Lecture 2 in formal language.
5. Prove Theorem 6 of the notes for Lecture 2.
6. Prove Theorems 7 and 8 of the notes for Lecture 2.