

MATHEMATICS H311 — FALL 2015

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HOMEWORK ASSIGNMENT NO. 2, DUE ON TUESDAY, SEPTEMBER 15

The following is a list of 13 problems that I strongly recommend for you to do. The problems labeled “TO HAND IN” are the ones you are asked to hand in as Homework assignment No. 2.

1. Book, Exercise 2.2.1 on page 47.
2. Book, Exercise 2.2.2 on page 47.
3. Book, Exercise 2.2.4 on pages 47-48.
4. **(TO HAND IN.)** Book, Exercise 2.2.6 on page 48.
5. **(TO HAND IN.)** Book, Exercise 2.3.1 on page 54.
6. **(TO HAND IN.)** Book, Exercise 2.3.3 on page 54.
7. **(TO HAND IN.)** Give a detailed proof, using the definition of limit of a sequence, that the sequence

$$(1, -1, 1, -1, 1, -1, \dots)$$

—that is, the sequence $((-1)^{n+1})_{n=1}^{\infty}$ —is divergent. (That is, you must prove that

(*) There does not exist $a \in \mathbb{R}$ such that $\lim_{n \rightarrow \infty} (-1)^{n+1} = a$.

For this purpose, you should rewrite Statement (*) in the form “for every $a \in \mathbb{R}$ there exists a positive real number ε such that ...”, and then prove it using the rules for proving sentences with quantifiers that you learned in Math 300.)

8. **(TO HAND IN.)** Find the limit

$$\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}).$$

(That is, figure out for which real number a it is true that

$$(0.1) \quad \lim_{n \rightarrow \infty} \left(\sqrt{n+1} - \sqrt{n} \right) = a,$$

and then prove Equation (0.1) rigorously using Definition 2.2.1.

9. Book, Exercise 2.3.6 on page 54.
10. Book, Exercise 2.3.7 on pages 54-55.
11. Book, Exercise 2.3.8 on page 55.
12. **(TO HAND IN.)** Book, Exercise 2.3.10 on page 55. (NOTE: One of the four statements in this problem contains a serious typo. You should find the typo first, explain why it is a typo, and then do the problem using the version with the typo corrected.)
13. **(TO HAND IN.)** Book, Exercise 2.3.11 on page 55.