MATHEMATICS H311 — FALL 2015

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HOMEWORK ASSIGNMENT NO. 3, DUE ON FRIDAY, SEPTEMBER 18

(Updated version, September 15, 2015)

The following is a list of 10 problems that I strongly recommend for you to do. The problems labeled **"TO HAND IN"** *are the ones you are asked to hand in as Homework assignment No. 3.*

This version differs fron the previous one in that I have removed Exercise 2.4.5 (pages 60-61) from the list of problems to hand in, because I decided instead to do that problem in class.

1. Book, Exercise 2.3.12 on page 55.
2. **(TO HAND IN.)** Book, Exercise 2.4.1 on pages 59-60.
3. **(TO HAND IN.)** Book, Exercise 2.4.2 on page 60.
4. Book, Exercise 2.4.5 on pages 60-61.
5. **(TO HAND IN.)** Book, Exercise 2.4.6 on page 61.
6. **(TO HAND IN.)** Book, Exercise 2.4.7 on page 61.
7. Book, Exercise 2.4.8 on page 61.
8. Book, Exercise 2.4.9 on page 61.
9. Book, Exercise 2.4.10 on pages 61-62
10. **(TO HAND IN.)** For an arbitrary real number ρ such that $0 \leq \rho < 1$,

    (a) Prove that the sequence $(n\rho^n)_{n=1}^{\infty}$ is bounded, by proving first, by induction on n, that the inequality $(1+\alpha)^n \geq 1 + n\alpha$ holds for every natural number n and for every real number α such that $\alpha \geq 0$, and then using this inequality by writing $\frac{1}{\rho} = 1 + \alpha$ to conclude that $\rho^n \leq \frac{1}{n\alpha}$.

(b) Prove that $\lim_{n\to\infty} n^k \rho^n = 0$ for every natural number k. (HINT: Let $\sigma = \sqrt[k+1]{\rho}$, so $\rho = \sigma^{k+1}$. Show first that $0 \leq \sigma < 1$, then use the result of Part (a) to conclude that the sequence $(n\sigma^n)_{n=1}^{\infty}$ is bounded, conclude from this that the sequence $(n^{k+1}(\sigma^{k+1})^n)_{n=1}^{\infty}$ is bounded, that is, that $(n^{k+1}\rho^n)_{n=1}^{\infty}$ is bounded, and finally multiply by $\frac{1}{n}$ and use the result of Exercise 2.3.9 (a).

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