MATHEMATICS H311 — FALL 2015

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HOMEWORK ASSIGNMENT NO. 5, DUE ON FRIDAY, OCTOBER 2

(Corrected version, September 27, 2015)

The following is a list of 14 problems that I strongly recommend for you to do. The problems labeled **"TO HAND IN"** *are the ones you are asked to hand in as Homework assignment No. 5.*

This version differs fron the previous one in that I have added two questions to Problem 11.

1. Book, Exercise 2.6.2 on page 70.

2. Book, Exercise 2.6.3 on page 70.

3. **(TO HAND IN.)** Book, Exercise 2.6.4 on page 70.

4. **(TO HAND IN.)** Book, Exercise 2.6.5 on page 70.

5. Book, Exercise 2.6.6 on pages 70-71.

6. Book, Exercise 2.7.1 on page 76.

7. **(TO HAND IN.)** Book, Exercise 2.7.2 on page 77.

8. **(TO HAND IN.)** Using the fact that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}\,,$$

find the value of the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}\,.$$

9. **(TO HAND IN.)** Book, Exercise 2.7.4 on page 77.

10. Book, Exercise 2.7.6 on page 77.

11. **(TO HAND IN.)** Prove or disprove each of the following four statements:

    i. If $(a_n)_{n=1}^{\infty}$ is a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} a_n$ diverges, then the series $\sum_{n=1}^{\infty} \frac{a_n}{1+na_n}$ converges.

    ii. If $(a_n)_{n=1}^{\infty}$ is a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} a_n$ diverges, then the series $\sum_{n=1}^{\infty} \frac{a_n}{1+n^2a_n}$ converges.

    iii. If $(a_n)_{n=1}^{\infty}$ is a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} a_n$ diverges, then the series $\sum_{n=1}^{\infty} \frac{a_n}{1+na_n}$ diverges.

    iv. If $(a_n)_{n=1}^{\infty}$ is a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} a_n$ diverges, then the series $\sum_{n=1}^{\infty} \frac{a_n}{1+n^2a_n}$ diverges.

12. **(TO HAND IN.)** Book, Exercise 2.7.7 on page 77.

13. Book, Exercise 2.7.8 on pages 77-78.

14. **(TO HAND IN.)** Book, Exercise 2.7.9 on page 78. (NOTE: The second statement of Part (a) says the following:
    "Explain why $(\exists N \in \mathbb{N})(\forall n \in \mathbb{N})(n \geq N \Longrightarrow |a_{n+1}| \leq |a_n|r')$.")