p.711 #2, 14, 16
p.717 #4, 10.

HW6 Solutions.

p.711 #2. $\frac{\partial^2 u}{\partial x^2} = -\frac{\partial^2 u}{\partial y^2}$ $\quad (0,a)\times(0,b)$

$u(0,y)=0,\ u(a,y)=0 \quad y\in(0,b)$

$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0,\ u(x,b)=f(x) \quad x\in(0,a)$

Solution: $u=\varphi(x)\psi(y)$ gives 2 equations.

① $\varphi''+\lambda\varphi=0,\ \varphi(0)=\varphi(a)=0$

② $\psi''-\lambda\psi=0,\ \psi'(0)=0.$

① $\lambda\le 0$ is not an eigenvalue due to the conditions $\varphi(0)=0=\varphi(a)$

If $\lambda=\alpha^2>0,\ \varphi(x)=A\cos\alpha x+B\sin\alpha x$

$\varphi(0)=A=0$

$\varphi(a)=0 \Rightarrow \alpha=\frac{n\pi}{a},\ n=1,2,3,\dots$

$\lambda_n=\frac{n^2\pi^2}{a^2},\ n=1,2,3,\dots$

$\varphi_n(x)=\sin\frac{n\pi x}{a}$

② $\lambda_n=\frac{n^2\pi^2}{a^2} \Rightarrow \psi(y)=A\cosh\frac{n\pi y}{a}+B\sinh\frac{n\pi y}{a}$

$\psi'(0)=\frac{n\pi B}{a}=0 \Rightarrow B=0$

so $\psi(y)=A\cosh\frac{n\pi y}{a}.$

so $u(x,y)=\sum_{n=1}^{\infty}A_n\cosh\frac{n\pi y}{a}\sin\frac{n\pi x}{a}$

$u(x,b)=\sum_{n=1}^{\infty}A_n\cosh\frac{n\pi b}{a}\sin\frac{n\pi x}{a}=f(x),\quad 0\le x\le a$