and $A_n = g \int_0^1 (1-x^2) \sin n\pi x \, dx$

$u = 1-x^2 \quad dv = \sin n\pi x$

$du = -2x\,dx \quad v = -\frac{1}{n\pi} \cos n\pi x$

$= g\left[\frac{x^2-1}{n\pi} \cos n\pi x \Big|_0^1 - \frac{2}{n\pi} \int_0^1 x \cos n\pi x \, dx \right]$

$u = x \quad dv = \cos n\pi x$

$du = dx \quad v = \frac{1}{n\pi} \sin n\pi x$

$= g\left[\frac{1}{n\pi} - \frac{2x}{n^2\pi^2} \sin n\pi x \Big|_0^1 + \frac{2}{n^2\pi^2} \int_0^1 \sin n\pi x \, dx \right]$

$= g\left[\frac{1}{n\pi} - \frac{2}{n^3\pi^3} \cos n\pi x \Big|_0^1 \right]$

$= g\left[\frac{1}{n\pi} - \frac{2}{n^3\pi^3} \left((-1)^n - 1\right) \right]$

$= \frac{g}{n\pi}\left[1 + \frac{2}{n^2\pi^2}\left(1-(-1)^n\right) \right].$

So $u(x,t) = \frac{g}{2}(x^2-1) + \frac{g}{\pi} \sum_{n=1}^{\infty} \left(\frac{n^2\pi^2 + 2(1-(-1)^n)}{n^2\pi^2} \right) \cos an\pi t \sin n\pi x.$