Math 250–Section #4 Hourly #1

Name: ____________________

1.[10 pts] Evaluate the determinant using cofactor expansions

$$\begin{vmatrix} 2 & 2 & -3 & 1 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 4 & 1 \\ 2 & 3 & 0 & 0 \end{vmatrix}$$

2. [12 pts] Find all the values for t for which the resulting system of equations (a) has no solution, (b) a unique solution, and (c) infinitely many solutions.

$$\begin{array}{rcrcrcl} x & + & y & - & z & = & 2 \\ x & + & 2y & + & z & = & 3 \\ x & + & y & + & (t^2-5)z & = & t \end{array}$$

3. [12 pts] Let V be the set of all 3×3 matrices with the format

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Show that V is a *subspace* of the vector space of all 3×3 matrices.

4. [10 pts] Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

5.[12 pts] (a) Explain what is a *linear combination* of vectors.
(b) Find out whether the vector $(1, 0, 1, 2)$ is a linear combination of $(2, 1, 3, 0)$, $(7, 3, 1, -1)$ and $(0, 1, 4, 3)$.

6. [12 pts] Let $u = (1, 2, -1)$ and $v = (3, -1, 1)$ be vectors of $\mathbb{R}^3$.
(a) Show that u is perpendicular [orthogonal] to v.
(b) Find a third nonzero vector $w = (a, b, c)$ perpendicular to both u and v.

7. [10 pts] If A, B and C are $n \times n$ matrices of determinants 2, 1 and 3 respectively. Explain the statements:
(a) The product ABC is invertible.

(b) The inverse of ABC is

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}.$$

8. [10 pts] Prove that the set of all polynomials $f(t)$, of degree at most 2, and such that $f'(1) = 0$ [$f'(t)$ denotes the derivative] is a subspace.

9. [12 pts] Using Cramer's rule, solve the system of equations

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{bmatrix} \cdot \mathbf{x} = \mathbf{b},$$

for 3 choices of $\mathbf{b}$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Further, if you use the solutions, respectively, as the columns of a 3×3 matrix B, how is it related to the matrix of the system?