Typicality, Preclusion and Quantum Measure Theory

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Sheldon Goldstein's Fest, Rutgers, October 7-9 2007

Plan of Talk

- Classical stochastic theories
- Quantum stochastic theories (Quantum Measure Theory)
- Typicality and Preclusion
- The Three Sit Experiment
- What is real in a Quantum Measure Theory?
- Non-classical Logic (Rules of Inference)
- Summary

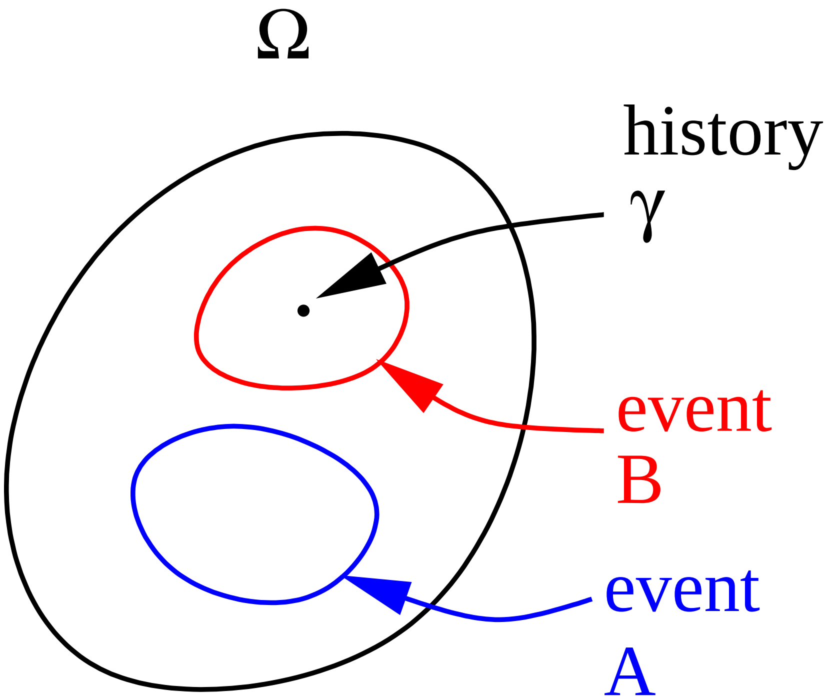
References:

- Rafael D. Sorkin (19??) “Quantum Mechanics as Quantum Measure Theory”
- Rafael D. Sorkin (2006) “Quantum Dynamics without the Wave Function” [quant-ph/0610204]
- Rafael D. Sorkin (2007) “An Exercise in Anhomomorphic Logic” [quant-ph/0703276]

A Common Structure

Every classical stochastic theory has the following structure:

- A sample space, Ω, of possible spacetime histories (e.g. sequences of outcomes of 1000 coin tosses, Weiner paths for a Brownian particle).
- A event algebra, $\mathfrak{A}$, whose elements are the possible questions that can be asked about the system.
- A measure, μ, on $\mathfrak{A}$ which encodes the dynamics and initial conditions.
- A single reality, which is one element γ of Ω, which contains the answer, yes or no, to every question that can be asked.



For example, Ω could be the set of all weather patterns and A could be the event "It is raining" and γ could be one weather pattern in which it is not raining. Each subset of Ω is an event/question. So $\mathfrak{A} = 2^{\Omega}$ and has a Boolean structure. (Ω finite).

If the event B is "It is dry and windy" and γ is the realised history then we answer the question "B?" with "Yes" and the question "A?" with "No."

The Role of the Measure

In a classical theory, the dynamics and initial state are encoded in a measure, μ which is a positive real function on the event algebra

$$\mu: \ \mathfrak{A} \rightarrow \mathbb{R}$$

$$\mu(A) \geq 0, \ \ \forall A \in \mathfrak{A}$$

$$\mu(A \sqcup B) = \mu(A) + \mu(B), \ \ \forall A, B \in \mathfrak{A}$$

$$\mu(\Omega) = 1$$

μ is interpreted as a probability measure: $\mu(A)$ is the probability of event A.

Quantum Theory from a Spacetime Perspective

Following Dirac and Feynman, we can express the formalism of quantum mechanics in fully spacetime terms: in terms of histories. In this form, quantum mechanics has much of the structure reviewed above:

- A sample space, Ω, of possible spacetime histories.
- A event algebra, $\mathfrak{A}$, whose elements are subsets of Ω.
- A measure, μ, on $\mathfrak{A}$ which encodes the dynamics and initial state.

The sample space is the set of histories that go into the Dirac-Feymnan Sum-Over-Histories. The measure $\mu(A)$ is less familiar: roughly speaking it is the mod squared of the sum of the quantum amplitudes of all the histories in A. (It is the diagonal term of the “decoherence functional.”)

This way of thinking about a quantum theory is not the usual one, but these structures exist for every quantum theory. And indeed they furnish an alternative foundation from which the usual Hilbert space can be constructed.

Can we also take over the axiom that One History Happens?

The Quantum Measure

The quantum measure μ is a positive real function on the event algebra (normalised)

$$\mu : \mathfrak{A} \to \mathbb{R}$$
$$\mu(A) \geq 0, \quad \forall A \in \mathfrak{A}$$
$$(\mu(\Omega) = 1)$$

but it does not, in general, satisfy the Kolmogorov sum rule, because of quantum interference:

$$\mu(A) = |\sum_{\gamma \in A} a(\gamma)|^2$$
$$\mu(A \sqcup B) = |\sum_{\gamma \in A \sqcup B} a(\gamma)|^2$$
$$= \mu(A) + \mu(B) + \text{ interference terms}$$

(μ does, however, satisfy a generalised sum rule for the disjoint union of three events.)

Typicality and Preclusion

This interference means that μ CANNOT be interpreted as a probability measure. How then are we to use it for scientific purposes? Going back to the classical case, we can ask the same question: how do we, in fact, use μ to do science?

Claim: We identify events E such that $\mu(E) << 1$ and we say, typically E does not happen: it is precluded.

This is known as Cournot's Principle, was first articulated by Bernoulli, occurs in Kolmogorov's "Grundbegriffe" as "Principle B", and is the interpretation of probability as applied to the real world held by Markov, Borel, Lévy and other founders of probability theory and by Shelly Goldstein.

Goldstein in his paper on Boltzmann

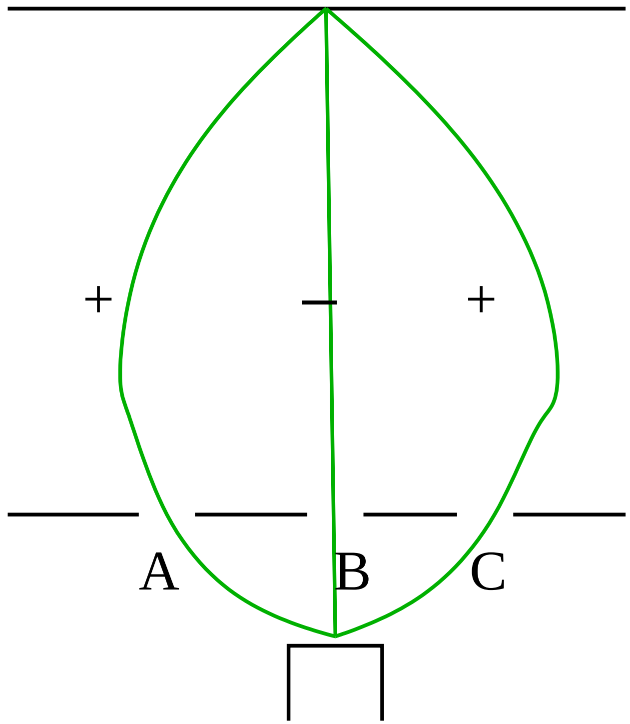
“With regard to ‘probabilistic’ or ‘statistical’ explanations, involving uncertainty about initial conditions, we say that a phenomenon has been explained if it holds for *typical* initial conditions, that is with rare exceptions as defined by a suitable ‘measure’ μ of typicality. The phenomenon has been explained if the set E of exceptional initial conditions satisfies $\mu(E) \ll 1$.

Of course it is essential that the measure of typicality be natural and not contrived. It should be an object that could, somehow, have been agreed upon before the phenomenon to be explained was even considered. For dynamical systems such as we are discussing here, the measure of typicality should be naturally related to the dynamics, [...].

Here is a small point, but one worth making if we intend to worry a bit about the justification of explanation via typicality: While typicality is usually defined – as it was here – in terms of a probability measure, the basic concept is not genuinely probabilistic, but rather a less detailed concept. A measure μ of typicality need not be countably additive, nor even finitely additive. Moreover, for any event E, if μ is merely a measure of typicality, there is no point worrying about, nor any sense to, the question as to the real meaning of say ‘$\mu(E) = 1/2$’. Distinctions such as between ‘$\mu(E) = 1/2$’ and ‘$\mu(E) = 3/4$’ are distinctions without a difference.”

The Three Slit Experiment

Let us adopt the Cournot-Goldstein Principle in Quantum Measure Theory.

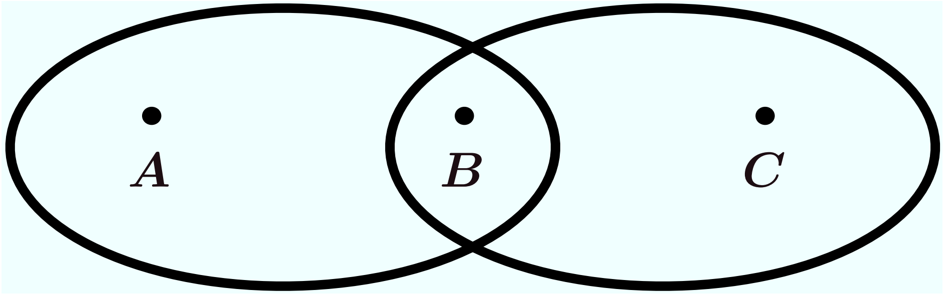


In this experiment, we can arrange the slits and choose a final position on the screen such that the amplitude for the particle to go through the middle slit B and land at the final position is -1 and the amplitude for the particle to go through the outer slit A is +1 and similarly for outer slit C.

The sample space consists of three histories, A, B and C (actually A contains many particle trajectories but this simplification preserves the essential point).

There are two events of measure zero: $\{A, B\}$ and $\{B, C\}$

No single history can happen



If one history γ happens and if an event has measure zero, then γ cannot be in that event.

BUT, the two sets of measure zero cover the whole sample space, so no single history can happen.

This example is not absolutely conclusive because there are other positions on the screen to consider (this is a subset of the full sample space). There are other conclusive examples, based on the Kochen-Specker theorem for example.

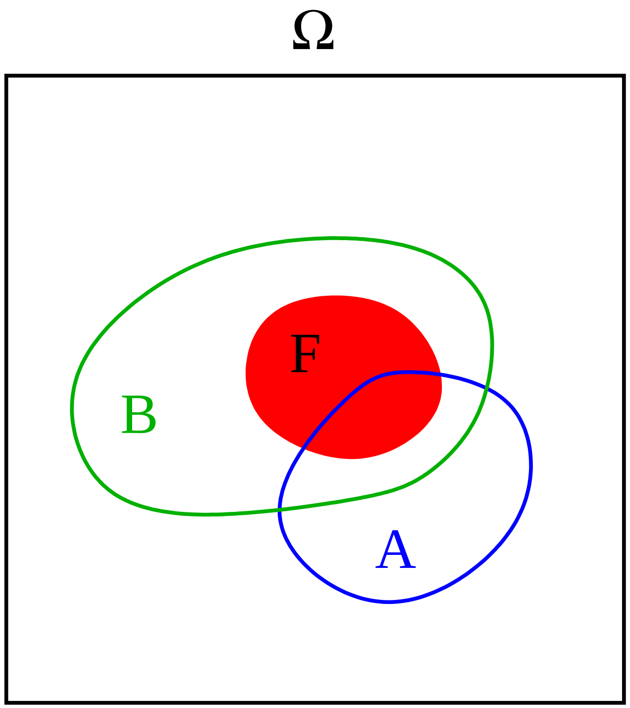
A Resolution

What do we replace the One History Axiom with? Suggestion (Sorkin):

"A subset of histories happens"

and

"That subset is minimal, subject to the preclusion condition"



The set of histories that happens is F. Question "Does event B happen?", Answer: yes. "Does event A happen?", Answer:no. In general if F is a subset of X then X is true, otherwise X is false. F must not be contained in a precluded event.

This "coarse graining" of reality gives non-Classical Logic.

The Un-noticed Background

Classical logic is a background assumption that is so basic we do not notice we are making it. “Logic” here means “Rules of inference regarding the truth and falsity of statements about the physical world”

The weather again:

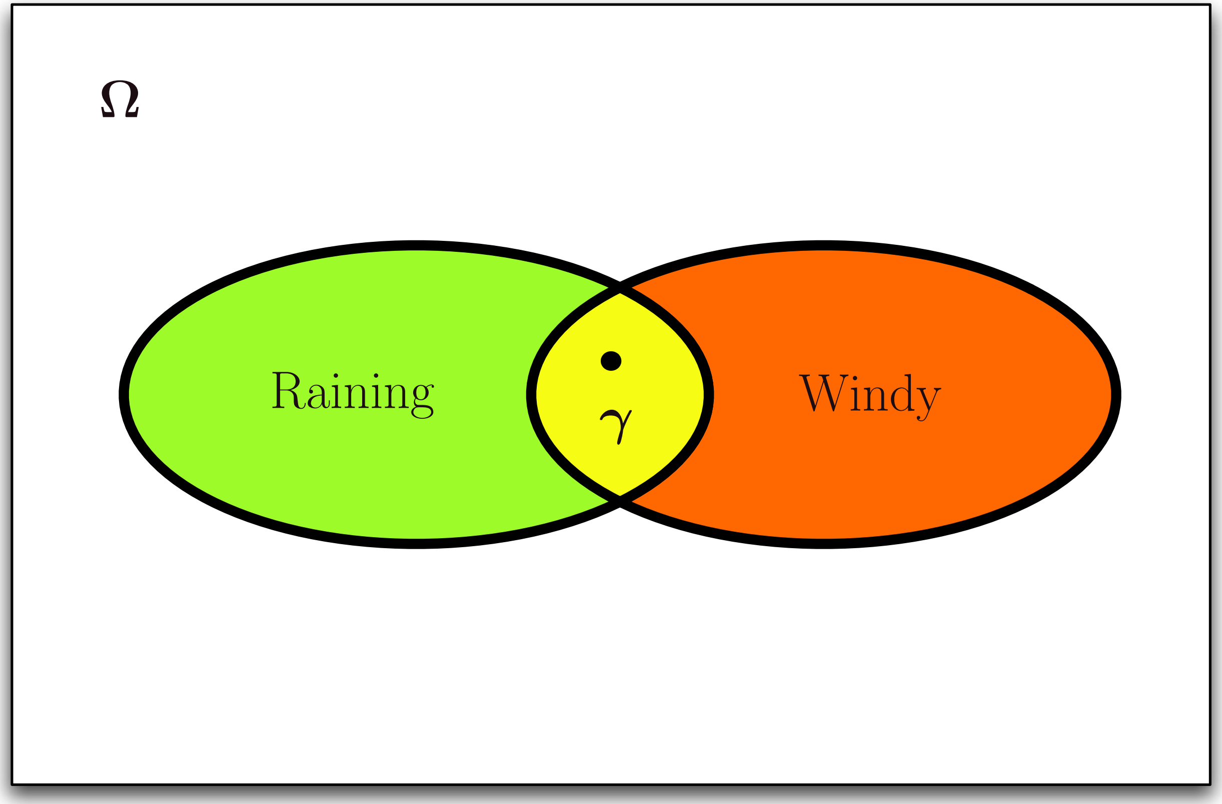
- If “it is raining” is true and “it is windy” is true then “it is raining and windy” is true
- If “it is raining” is true then “it is dry” is false
- If “it is raining” is false then “it is dry” is true

Classical Logic $\equiv$ One History Happens

CLAIM: We use classical logic as an unquestioned background because, in everyday life,

One History Happens.

Wind and Rain

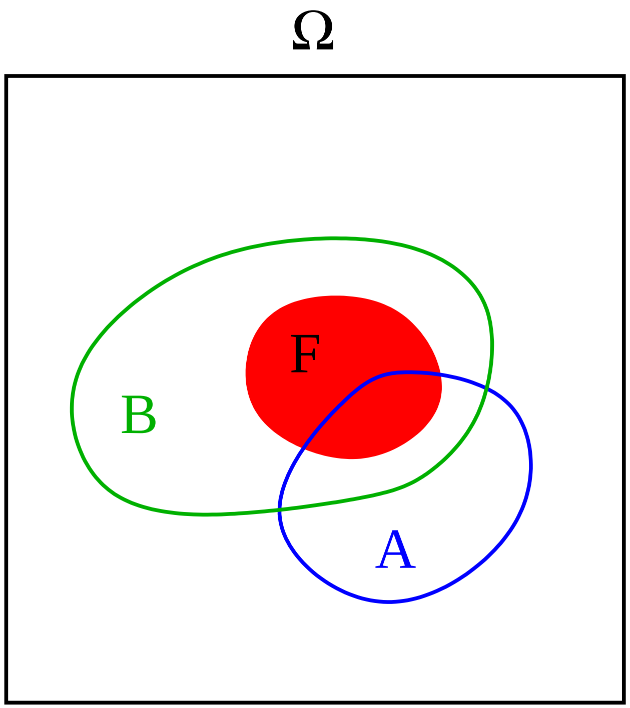


- If "it is raining" is true and "it is windy" is true then "it is raining and windy" is true
- If "it is raining" is true then "it is dry" is false
- If "it is raining" is false then "it is dry" is true

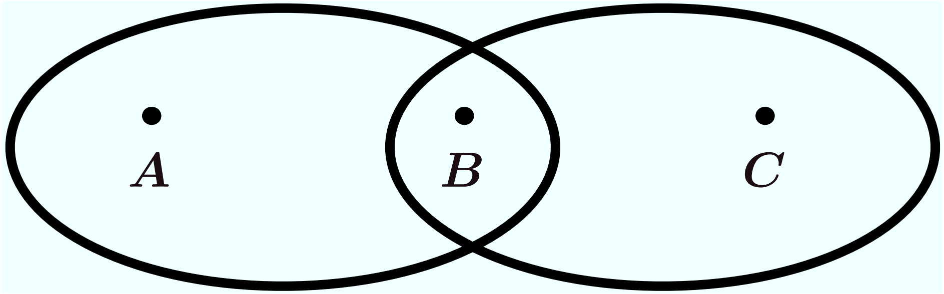
Coarse grained Reality $\equiv$ Non-Classical Logic

If what happens is a set of histories, let's look again at our rules of inference:

- If "R" is true and "W" is true then "R and W" is true HOLDS
- If "R" is true then "not R" is false HOLDS
- If "R" is false then "not R" is true FAILS



Three Slits revisited



The real set of histories cannot be contained in a set of measure zero. So it cannot be a subset of $\{A, B\}$ or $\{B, C\}$. There are only two possible sets satisfying this condition: $\{A, C\}$ and the whole set $\{A, B, C\}$. The latter is not minimal because it contains the former. So the unique minimal preclusive reality is $\{A, C\}$. We can answer any question: e.g.

Does the particle go though slit A? No

Does the particle go through slit B? No

Does the particle go through slit C? No

Does the particle go through slit A or B? No

Does the particle go through slit A or C? Yes

Real properties are common properties of all the histories in the “real set”. This is a true coarse graining – finer details are unphysical.

Summary and Comments

- Sum-over-histories quantum mechanics shares much of the same framework as classical stochastic theories: sample space, event algebra, measure.
- Quantum interference and the law that events of very small measure typically do not happen means that the One History axiom must be given up.
- Replaced by "A set of histories happens" and "maximal detail is achieved in reality"
- Classical rules of inference are a consequence of the One History Happens axiom of classical physical theories.
- With the new axioms, classical logic is recovered when the measure is classical (maximal detail drives the "real set" to be a singleton).
- For viability, we need to show that, in a fundamentally quantum theory, classical logic is recovered for macroscopic events.

Happy Birthday Shelly!