On the Present Status of Quantum Mechanics

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Schrodinger's Cat

(2) If geiger counter is triggered, hammer falls
(1) Radioactive material has a 50:50 chance of triggering geiger counter
(3) Hammer breaks poison bottle
(4) Cat dies if poison bottle breaks
(5) Cat lives, if Geiger counter does not trigger hammer and releases the poison
What does the cat example mean?

- It’s often called a “paradox,” but that is too weak—that sounds like “get used to it.”
- Basically, it’s an argument: Cat + atom belong to a quantum system of $10^{25}$ electrons, protons and neutrons, with a wave function $\psi$ governed by the Schrödinger equation. Since the Schrödinger equation is linear, we have that, after 1 hour, the wave function is a “superposition” of the wave function of a dead cat and that of a live cat:

$$\psi = \psi_{\text{dead}} + \psi_{\text{alive}}.$$ 

However, in reality the cat must be either dead or alive.

John S. Bell: “The problem is: AND is not OR.”

Also known as “the measurement problem of quantum mechanics.”
Consider a quantum measurement of the observable \( A = \sum_n \alpha_n |n\rangle \langle n| \).

\[
|n\rangle \otimes \phi_0 \xrightarrow{t} |n\rangle \otimes \phi_n
\]

(\( \phi_0 = \) ready state of apparatus, \( \phi_n = \) state displaying result \( \alpha_n \))

\[
\Rightarrow \sum_n c_n |n\rangle \otimes \phi_0 \xrightarrow{t} \sum_n c_n |n\rangle \otimes \phi_n
\]

But one would believe that a measurement has an actual, random outcome \( n_0 \), so that one can ascribe the “collapsed state” \( |n_0\rangle \) to the system and the state \( \phi_{n_0} \) to the apparatus.

**Conclusion from this argument:**

- Either \( \psi \) is not the complete description of the system,
- or the Schrödinger equation is not correct for \( N > 10^{20} \) particles,
- or there are many worlds.
Foundations of QM: goals

- resolve the paradoxes of QM
- obtain an explanation of QM
- provide a theory about the foundations of QM that can be understood as clearly as classical mechanics—a “quantum theory without observers” (K. Popper)
- use this theory as a clean mathematical basis to turn “folklore” knowledge into theorems
- make QM easier to learn
- fight mysticism about QM, unwarranted claims, and bad philosophy
Foundations of QM: goals

- obtain an explanation of QM

Example. To be explained: outcome of double-slit experiment

A possible explanation: Bohmian trajectories
- quantum formalism: axioms about observations
- theory about foundations: axioms without observers
When do you need the foundations of QM?

- in principle
- practical
When do you need the foundations of QM?

- in principle: When you want to *understand* QM.
- practical: When you want to *analyze* measurements.
When do you need the foundations of QM?

- in principle: When you want to *understand* QM.

  Although a dose of practically-mindedness is helpful, in the end of the day we all set a high value on genuine understanding.

- practical: When you want to *analyze* measurements.
When do you need the foundations of QM?

- in principle: When you want to understand QM.
- practical: When you want to analyze measurements.

Example

You want to prove a superselection rule for a particular system (i.e., that a self-adjoint operator $A$ will not be an observable unless it commutes with a certain operator $S$).
Bohr’s notion of “complementarity” was not helpful.
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Insist on the highest standards of quality.
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Insist on the highest standards of quality.

Don’t buy any theory that’s weird. Don’t accept that “it’s normal for quantum theories not to make sense.” Quantum theories without observers can and should be as clearly understandable as classical mechanics.
Bohr’s notion of “complementarity” was not helpful.

Insist on the highest standards of quality.

The problem is not lack of existence, it’s lack of uniqueness.
Morals of research in foundations of QM

1. Bohr’s notion of “complementarity” was not helpful.
2. Insist on the highest standards of quality.
3. The problem is not lack of existence, it’s lack of uniqueness.

Bohr believed that no quantum-theories-without-observers exist. That’s wrong. Several exist:

Theories that work
- Bohmian mechanics
- spontaneous collapse
- many worlds (in Schrödinger’s version, rather than Everett’s)

Research programs that may (one day) lead to theories that work
- “quantum measure theory” [R. Sorkin, F. Dowker]
- decoherent histories/consistent histories [M. Gell-Mann, J. Hartle, R. Omnès, R. Griffiths]
Bohmian mechanics

- takes the word “particle” literally: The $k$-th particle has position $Q_k(t) \in \mathbb{R}^3$ at time $t$.
- The complete description of a system is $(Q_1, \ldots, Q_N, \psi)$.
- The equation of motion is the simplest possible:

\[
\frac{dQ_k}{dt} = \text{probability current}\cdot\text{probability density}
\]

with current $= (\hbar/m_k) \text{Im} \psi^* \nabla_k \psi$

and density $= |\psi|^2$.

- $\psi$ evolves according to the Schrödinger eq.

\[
i\hbar \frac{\partial \psi}{\partial t} = H\psi
\]

Like it or don’t, it actually works: Inhabitants of a universe governed by Bohmian mechanics (with $|\psi|^2$-distributed initial configuration) would observe exactly the probabilities predicted by the quantum formalism.
Bohmian mechanics

Wave–particle dualism in the literal sense: there is a wave, and there is a particle. The path of the particle depends on the wave.

Shown: A double-slit and 80 possible paths of Bohm’s particle. The wave passes through both slits, the particle through only one.
Spontaneous collapse

e.g., the “GRW flash theory” (GRWf):

Instead of Bohm’s world lines, there are world points in space-time, called “flashes.” A macroscopic object consists of a galaxy of flashes.
Spontaneous collapse: GRW theory

Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles \((N < 10^4)\) but not for macroscopic systems \((N > 10^{23})\). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger’s cat) of macroscopically different contributions.

Put differently, regard the collapse of \(\psi\) as a physical process governed by mathematical laws.

Explicit equations by Ghirardi, Rimini, and Weber (1986)

The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.
GRW theories are empirically adequate. Their predictions deviate very very slightly from the quantum formalism.

Parameter diagrams (log-log scale). ERR = empirically refuted region, PUR = philosophically unsatisfactory region.

[Feldmann, Tumulka 2012]
GRW’s stochastic evolution for $\psi$

- is designed for non-relativistic quantum mechanics of $N$ particles
- meant to replace Schrödinger eq as a fundamental law of nature
- involves two new constants of nature:
  - $\lambda \approx 10^{-15} \text{ sec}^{-1}$, called collapse rate per particle.
  - $\sigma \approx 10^{-7} \text{ m}$, called collapse width.
- Def: $\psi$ evolves as if an observer outside the universe made, at random times with rate $N\lambda$, quantum measurements of the position observable of a randomly selected particle with inaccuracy $\sigma$.
- “rate $N\lambda$” means that $\text{prob(}\text{an event in the next } dt \text{ seconds)} = N\lambda \, dt$.
- more explicitly: Schrödinger evolution interrupted by jumps of the form
  $$\psi_{T^+} = e^{-\frac{(q_k-q)^2}{4\sigma^2}} \psi_{T^-},$$
  i.e., multiplication by a Gauss function with random label $k$, center $q$ and time $T$.
- A flash occurs at $(T, q)$ for each collapse.
GRW’s spontaneous collapse

before the “spontaneous collapse”:

and after:
Many worlds

Schrödinger proposed this theory in 1925:

Matter is distributed continuously in space with density

\[ m(q, t) = \sum_{k=1}^{N} \int_{\mathbb{R}^{3N}} \delta^3(q - q_k) |\psi_t(q)|^2 \, dq. \]

\( \psi_t \) evolves according to the Schrödinger eq.

He soon abandoned this theory because he thought it made wrong predictions. But actually, it is a many-worlds theory making right predictions: it implies the quantum formalism.

For Schrödinger’s cat, \( \psi = \frac{1}{\sqrt{2}} \psi_{\text{dead}} + \frac{1}{\sqrt{2}} \psi_{\text{alive}} \), it follows that \( m = \frac{1}{2} m_{\text{dead}} + \frac{1}{2} m_{\text{alive}}. \)

There is a dead cat and a live cat, but they are like ghosts to each other (they do not notice each other), as they do not interact. So to speak, they live in parallel worlds.
Not knowing about Schrödinger’s proposal, Everett advocated a many-worlds view in 1957, but with an unclear (or inadequate) ontology: His idea was that for wave functions such as Schrödinger’s cat’s, both cats are in the wave function, so both cats exist.

Everett contributed substantially to the analysis of probabilities in a many-world framework.

Both Schrödinger’s and Everett’s many-worlds theory have serious difficulty explaining why we see $|\psi|^2$ frequencies, and may therefore ultimately not be viable.  

[Kent 2010, Allori et al. 2011]
The measurement problem is solved

Conclusion from the quantum measurement problem:

1. Either $\psi$ is not the complete description of the system,
2. or the Schrödinger equation is not correct for $N > 10^{20}$ particles,
3. or there are many worlds.

- This is the case in Bohmian mechanics, as the complete description is $(\psi, Q)$.
- This is the case in collapse theories such as GRW.
- Many worlds.
Bohr’s notion of “complementarity” was not helpful.

Insist on the highest standards of quality.

The problem is not lack of existence, it’s lack of uniqueness.

The ultimate arbiter between quantum theories without observers will be quantum gravity.
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Get used to limitations to knowledge and control.
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**Limitation to knowledge**

means that there is a fact in the world but we can’t find it out empirically, or a well-defined variable that we can’t measure.

Give up the idea that a scientific theory should involve only observable quantities. That’s exaggerated positivism.

**John Bell (1987):**

“To admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics.”

**Example:** The wave function is not an observable quantity. If you prepare a particle with wave function \( \psi \), no experiment I can do will reveal to me what \( \psi \) is. Nature can keep a secret, and QM proves it.
Get used to limitations to knowledge and control.

**Limitations to knowledge in quantum-theories-without-observers:**

Some people don’t like Bohmian mechanics because you can’t observe the entire trajectory without disturbing it. Not a good reason.

Some people don’t like many-worlds theories because you can’t observe the other worlds. Not a good reason.

Inhabitants of a hypothetical universe governed by the GRW collapse theory can’t detect reliably (only with limited probability) how many collapses occurred in a given system in a given time interval.

[Cowan and R.T. 2013]
Get used to limitations to knowledge and control.

**Limitation to control**

means that there is a physical variable that we can’t push to a desired value.

In Bohmian mechanics, the configuration of a system is random with probability density $|\psi|^2$, and we can’t choose it, i.e., we can’t manipulate it to be a specific configuration without changing $\psi$. 
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“Ontology” is an important concept.
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Ontology of a theory = what exists physically, according to that theory.

Jeremy Bernstein (2011):

“Many of the papers I tried to read were written by philosophers and had words like “ontology” sprinkled over them like paprika.”

It was long thought that the key to clarity in QM was to avoid talking about ontology and stick to operational statements. That thought has not paid off.
“Ontology” is an important concept.

Be clear about what is real.

**ontology of Bohmian mechanics:**
particles and $\psi$

**ontology of GRWf:**
flashes and $\psi$

**ontology of Schrödinger’s many-worlds:**
continuous distribution of matter with density $m(q, t)$ and $\psi$
Bohr’s notion of “complementarity” was not helpful.

2. Insist on the highest standards of quality.

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4. The ultimate arbiter between quantum theories without observers will be quantum gravity.

5. Get used to limitations to knowledge and control.

6. “Ontology” is an important concept.

7. Don’t take the word “measurement” literally.

Literally, to “measure” means to reveal a quantity that was defined already before the experiment. Quantum experiments are usually not like that. To pretend they were (i.e., to adopt a naive realism about operators) leads to unnecessary weirdness and paradoxes.
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Don’t take the word “measurement” literally.

Not all observables are created equal: position is more important.

All 3 quantum theories without observers (Bohmian mechanics, collapse à la GRW, many-worlds à la Schrödinger) give special status to position operators.
Quantum non-locality

Bell’s theorem (1964)

Although it is not possible to send messages faster than light, sometimes events $A$, $B$ occurring at spacelike separation (i.e., so that no light signal can travel from one to the other) must influence each other. We do not know whether $A$ influences $B$ or vice versa.

Bell’s theorem is often confused with Bell’s lemma: If there were variables pre-determining the outcome of every experiment then they could be influenced faster than light.

Bell’s lemma + EPR argument $\Rightarrow$ Bell’s theorem

All 3 quantum theories without observers (Bohmian mechanics, collapse à la GRW, many-worlds à la Schrödinger) are nonlocal.
All 3 quantum theories without observers have been extended to non-relativistic quantum field theory:

- For Bohmian mechanics, there are two approaches:
  - Field ontology \( (\phi(q), \Psi[\phi]) \), works only for bosonic fields [Bohm 1952; Struyve 2010]
  - Particle ontology with particle creation and annihilation [Bell 1986; Dürr, R.T., et al. 2004]

- GRW theory [Pearle, Ghirardi et al. 1990; R.T. 2005]
- Schrödinger’s many-worlds theory [Allori, R.T., et al. 2011]
Extensions to relativity

Challenge: Devise a theory that is nonlocal and relativistic

- Bohmian mechanics: If a preferred slicing of space-time into spacelike hypersurfaces ("time foliation") is granted, then there is a simple, convincing analog of Bohmian mechanics. [Dürr et al. 1999]

- Also a GRW-style theory can be easily defined using a time foliation.

- But GRW can be made "more relativistic"—without a time foliation
  
  This has been done for \( N \) non-interacting particles, including entanglement/nonlocality [R.T. 2006]

- Pearle’s [1990] model includes interaction without a time foliation but is mathematically badly defined (divergent).

- Bedingham’s [2011] model includes interaction and is well defined

- Schrödinger’s many-worlds theory is easily made relativistic:
  \[
  m_{\mu\nu}(x^\lambda) = \langle \psi | T_{\mu\nu}(x^\lambda) | \psi \rangle \text{ with } T_{\mu\nu} = \text{energy-momentum operator.}
  \]
The main obstacle to devising an analog of Bohmian mechanics or GRWf theory for (say) full QED is that it is not clear which position operators to use.

Equivalently, which probability density $\rho$ in position representation.

For a photon, the wave function $\psi$ is mathematically equivalent to a classical Maxwell field $(E, B)$. Landau and Peierls (1930) proposed $\rho = \tilde{E}^2 + \tilde{B}^2$ with $\tilde{\psi} = F^{-1} \omega(k)^{-1/2} F$, but this rule is nonlocal in $E$ and $B$ ($\Rightarrow$ superluminal signaling) and not Lorentz covariant.

For an electron, $\psi = (\psi_1, \ldots, \psi_4)$ is a Dirac wave function and $\rho = \sum_s |\psi_s|^2$. But electron-positron pair creation at location $x$ would seem to lead to superluminal signaling over the distance given by the width of the positive-energy part of a Dirac $\delta$ function.

Widespread view: There are no position operators.

But we can detect photons. So what replaces the position operators? That is an open question for all interpretations of quantum theory.
Thank you for your attention