The Metaphysics of Quantum Mechanics

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For doing physics, one needs

- experiments
- mathematics
- philosophy
For doing physics, one needs
- experiments

We want theories that are empirically adequate.
- mathematics
- philosophy
For doing physics, one needs

- experiments
- mathematics
- vectors, tensors, differential equations, ...
- philosophy
For doing physics, one needs
- experiments
- mathematics
- philosophy

What kinds of theories are acceptable? Which kinds of explanations count as satisfactory?
Quantum mechanics
Quantum mechanics

- is very successful in computing quantities than can be compared to experimental data: spectral lines, energies of chemical bonds, interference patterns, ...

- What we know: We have rules for predicting the outcome of any experiment—the quantum formalism.

- What is controversial: What actually happens inside atoms, or in quantum experiments.
The quantum formalism, part 1

(rules for predicting the outcome of any experiment)
A system of $N$ electrons/protons/neutrons has a wave function

$$\psi_t : \mathbb{R}^{3N} \to \mathbb{C}$$

$$\psi_t(q) = \psi_t(q_1, \ldots, q_N)$$

As long as the system is isolated, the wave function changes with time according to the Schrödinger equation

$$i\hbar \frac{\partial \psi_t}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla^2 \psi_t + V \psi_t,$$

which determines $\psi_t$ at all times $t$ if the “initial” wave function $\psi_{t_0}$ at any time $t_0$ is given.
“Measurement postulate”:

- Suppose we let the system interact with a macroscopic apparatus at time $t$.
- Formula for the probability that the apparatus obtains the outcome $z$:

$$\text{Prob}(z) = \int |P_z \psi_t(q)|^2 \, dq.$$  

$P_z$ are appropriate projection operators associated with this type of apparatus.

- $A = \sum_z zP_z$ is a self-adjoint operator, called “the observable.”

- “Collapse of the wave function”: After the interaction, $\psi$ must be replaced by $P_z \psi$. 

Example

- system of $N = 1$ particle
- apparatus = detector
- outcome = position of the particle
- $\text{Prob}(q) = |\psi_t(q)|^2$
- Collapse:

before:

after:
Brightness = \(|\psi|^2\), color = phase. Pictures: B. Thaller.

Let a wave function of wave length \(\lambda\) hit a screen with two narrow slits (width \(\approx\) \(\lambda\)). Due to interference (i.e., partial cancellation of waves), the wave passing through will show alternating fringes of brightness and darkness—an “interference pattern” (consequence of the Schrödinger equation).

Behind the double-slit, let the wave function hit a detector screen: we obtain a random spot with probability distribution \(|\psi(q)|^2\).
Repeat the experiment many times to obtain many spots on the detection screen with distribution $|\psi|^2$. 
The traditional “orthodox” view:

One should not ask the question “What happens?”.

Werner Heisenberg in 1958:

“The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them […], is impossible.”

“We can no longer speak of the behavior of the particle independently of the process of observation.”

W. Heisenberg (1901–1976)
Feynman in 1959:

“Does this mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the thought that without observer there is no reality? Which observer? Any observer? Is a fly an observer? Is a star an observer? Was there no reality before $10^9$ B.C. before life began? Or are you the observer? Then there is no reality to the world after you are dead? I know a number of otherwise respectable physicists who have bought life insurance.”

Richard Feynman (1918–1988)
Bell in 1990:

“It would seem that the theory is exclusively concerned about 'results of measurement', and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of 'measurer'? Was the world wave function waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some more highly qualified measurer—with a Ph.D.?”
Bohmian mechanics

An alternative to the orthodox view of QM
Bohmian mechanics

- takes the word “particle” literally: The $k$-th particle has position $Q_k(t) \in \mathbb{R}^3$ at time $t$.
- The complete description of a system is $(Q_1, \ldots, Q_N, \psi)$.
- The equation of motion is the simplest possible:
  \[
  \frac{dQ_k}{dt} = \text{probability current} / \text{probability density}
  \]
  with current $= (\hbar/m_k) \text{Im} \psi^* \nabla_k \psi$
  and density $= |\psi|^2$.
- $\psi$ evolves according to the Schrödinger eq.
  \[
  i\hbar \frac{\partial \psi}{\partial t} = H\psi
  \]
- The initial configuration $(Q_1(0), \ldots, Q_N(0))$ is typical relative to the $|\psi(0)|^2$ distribution, i.e., looks as if chosen randomly with $|\psi(0)|^2$ distribution.
Like it or don’t, it actually works: Inhabitants of a universe governed by Bohmian mechanics would observe exactly the probabilities predicted by the quantum formalism.
Werner Heisenberg in 1958:

“The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them [...], is impossible.”

“We can no longer speak of the behavior of the particle independently of the process of observation.”

Bohmian mechanics is a counter-example to the impossibility claim.
Bohmian mechanics

Wave–particle dualism in the literal sense: there is a wave, and there is a particle. The path of the particle depends on the wave.

Shown: A double-slit and 80 possible paths of Bohm’s particle. The wave passes through both slits, the particle through only one.
Most paths arrive where $|\psi|^2$ is large—that’s how the interference pattern arises. If one slit gets closed, the wave passes through only one slit, which leads to different trajectories and no interference pattern. The path of the particle depends on the wave.
Quantum non-locality

Bell’s nonlocality theorem (1964)

Although it is not possible to send messages faster than light, sometimes events $A$, $B$ occurring at spacelike separation (i.e., so that no light signal can travel from one to the other) must influence each other. We do not know whether $A$ influences $B$ or vice versa.

Bell’s theorem is often confused with Bell’s lemma: If there were variables pre-determining the outcome of every experiment then they could be influenced faster than light.
The logic of Bell’s proof

Bell’s theorem (1964)

Certain statistics of outcomes (predicted by QM) are possible only if spacelike separated events sometimes influence each other. (No matter which interpretation of QM is right.)

These statistics were confirmed in experiment [Aspect 1982 etc.].

Bell’s lemma (1964)

Non-contextual hidden variables are impossible in the sense that they cannot reproduce the statistics predicted by QM for certain experiments.

Upshot of Einstein-Podolsky-Rosen’s (EPR’s) argument (1935)

Assume that influences between spacelike separated events are impossible. Then there must be non-contextual hidden variables for all local observables.

Note: EPR + Bell’s lemma \(\Rightarrow\) Bell’s theorem

see also my arXiv:1501.04168
\[
\frac{dQ_1}{dt} \text{ depends on } Q_2(t), \text{ no matter the distance } |Q_1(t) - Q_2(t)|.
\]
Bohmian mechanics in relativistic space-time

- If a preferred foliation (= slicing) of space-time into spacelike hypersurfaces ("time foliation" \( \mathcal{F} \)) is permitted, then there is a simple, convincing analog of Bohmian mechanics, \( \text{BM}_\mathcal{F} \).


Without a time foliation, no version of Bohmian mechanics is known that would make predictions anywhere near quantum mechanics. (And I have no hope that such a version can be found in the future.)
There is no agreed-upon definition of “relativistic theory.” Anyway, the possibility seems worth considering that our universe has a time foliation.

**Simplest choice of time foliation** \( \mathcal{F} \)

Let \( \mathcal{F} \) be the level sets of the function \( T : \text{space-time} \rightarrow \mathbb{R} \), \( T(x) = \text{timelike-distance}(x, \text{big bang}) \).

E.g., \( T(\text{here-now}) = 13.7 \text{ billion years} \)

Alternatively, \( \mathcal{F} \) might be defined in terms of the quantum state vector \( \psi \), \( \mathcal{F} = \mathcal{F}(\psi) \) [Dürr, Goldstein, Norsen, Struyve, Zanghì arXiv:1307.1714]

Or, \( \mathcal{F} \) might be determined by an evolution law (possibly involving \( \psi \)) from an initial time leaf.
Predictions

The detected configuration is $|\psi_\Sigma|^2$-distributed on every spacelike $\Sigma$. No superluminal signaling.

As a consequence,

$\mathcal{F}$ is invisible, i.e., experimental results reveal no information about $\mathcal{F}$. (A limitation to knowledge.)
Although it may seem to go against the spirit of relativity, I take seriously the possibility that our world might have a time foliation. However, there do exist relativistic realist theories of quantum mechanics that do not require a time foliation: A relativistic version of the Ghirardi-Rimini-Weber (GRW) collapse theory. [Tumulka arXiv:quant-ph/0406094]

The theory is somewhat more complicated and less natural than Bohmian mechanics.
Spontaneous collapse: GRW theory

Key idea:

The Schrödinger equation is only an approximation, valid for systems with few particles ($N < 10^4$) but not for macroscopic systems ($N > 10^{23}$). The true evolution law for the wave function is non-linear and stochastic (i.e., inherently random) and avoids superpositions (such as Schrödinger’s cat) of macroscopically different contributions.

Put differently, regard the collapse of $\psi$ as a physical process governed by mathematical laws.

Explicit equations by Ghirardi, Rimini, and Weber (1986)

The predictions of the GRW theory deviate very very slightly from the quantum formalism. At present, no experimental test is possible.
GRW theories are empirically adequate

Parameter diagrams (log-log scale). ERR = empirically refuted region, PUR = philosophically unsatisfactory region.

[Feldmann, Tumulka arXiv:1109.6579]
GRW’s stochastic evolution for $\psi$

- is designed for non-relativistic quantum mechanics of $N$ particles
- meant to replace Schrödinger eq as a fundamental law of nature
- involves two new constants of nature:
  - $\lambda \approx 10^{-15} \text{ sec}^{-1}$, called collapse rate per particle.
  - $\sigma \approx 10^{-7} \text{ m}$, called collapse width.
- **Def:** $\psi$ evolves as if an observer outside the universe made, at random times with rate $N\lambda$, quantum measurements of the position observable of a randomly selected particle with inaccuracy $\sigma$.
- “rate $N\lambda$” means that
  $\text{prob(}\text{an event in the next } dt \text{ seconds)} = N\lambda \ dt$.
- more explicitly: Schrödinger evolution interrupted by jumps of the form
  $$\psi_{T+} = e^{-\frac{(q_k - q)^2}{4\sigma^2}} \psi_{T-},$$
i.e., multiplication by a Gauss function with random label $k$, center $q$ and time $T$.  

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GRW’s spontaneous collapse

before the “spontaneous collapse”:

and after:
But the stochastic evolution law for $\psi$ is only half of the theory...
Primitive ontology
"Ontology” is an important concept

Ontology of a theory = what exists physically, according to that theory.

Physicist Jeremy Bernstein (2011):

“Many of the papers I tried to read were written by philosophers and had words like “ontology” sprinkled over them like paprika.”

It was long thought that the key to clarity in QM was to avoid talking about ontology and stick to operational statements. That thought has not paid off.
Several ontologies

- Suppose a theory $T$ talks about particles in the literal sense, having world lines in space-time.
  - Then we say that $T$ has a particle ontology.
  - Examples: Classical mechanics, Bohmian mechanics.
- Now suppose that a theory $T'$ says that matter is continuously distributed in 4D space-time, with density function $m(t, x)$ [or $m_\mu(t, x)$ or $m_{\mu\nu}(t, x)$].
  - Then we say that $T'$ has a matter density ontology.
Flash ontology
ontology of a theory = what exists, according to that theory

primitive ontology (PO) = the part of the ontology representing matter in 4D space-time

Example: In Bohmian mechanics,
ontology = \((Q, \psi)\),
primitive ontology = \(Q\),
non-primitive ontology = \(\psi\).
### Def: GRWf [Bell 1987]

If $\psi$ collapses at time $T$ with center $X$ then put a flash at $(T, X)$.

### Def: GRWm [Diósi 1989; Ghirardi, Grassi, Benatti 1995; Goldstein 1998]

Matter density given by (in the non-relativistic case)

$$m(t, q) = \sum_{k=1}^{N} m_k \int \delta^3(q - q_k) |\psi_t(q_1, \ldots, q_N)|^2 d^3q_1 \cdots d^3q_N$$

$$= \langle \psi_t | \mathcal{M}(x) | \psi_t \rangle$$

with $\mathcal{M}(x) = \sum_{k=1}^{N} m_k \delta^3(x - \hat{Q}_k)$ the mass density operators.


### Def: GRW0 (bad idea)

No PO, just wave function
Consider an EPR experiment, in which two particles in the singlet spin state are widely separated in space, and a Stern–Gerlach experiment is carried out on each particle. The reduced spin state $\rho$ of particle A (obtained by tracing out the spin of particle B) will depend on the choice of hypersurface $\Sigma$: If $\Sigma = \Sigma_2$ lies after the experiment on particle B but before that on particle A, then $\rho$ will be a pure state. If $\Sigma = \Sigma_1$ lies before both experiments, $\rho$ will be mixed.

This poses a problem of finding a consistent relativistic specification of facts for GRW0. However, the problem evaporates for GRWf/m.
**Toy theories**

[Allori, Goldstein, Tumulka, Zanghì arXiv:1206.0019]

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**GRWp₁:**

PO = particles; Bohm's eq of motion for $Q$; GRW evolution for $\psi$.  
Predictions: neither equivalent to Bohm nor to GRW; empirically wrong.  
Schrödinger's cat may be alive, then a collapse to $|\text{dead}\rangle$ occurs, but cat is still alive; then crazy behavior.  
**Upshot:** We need to know the PO for deriving predictions.

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**GRWp₂:**

PO = particles; Bohm's eq of motion for $Q$; GRW evolution for $\psi$ but with collapse center $X$ not random but $X = Q_k(T)$.  
$|\psi|^2$ not preserved $\Rightarrow$ empirically wrong.

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**GRWp₃:**

PO = particles; Bohm’s eq of motion for $Q$; GRW evolution for $\psi$ but with $X = Q_k(T) + Z$ with $Z$ independent 3D Gaussian random variable with mean 0 and variance $\sigma^2 I$.  
$|\psi|^2$ preserved; empirically equivalent to GRWm and GRWf.
How metaphysics ties in with the physics
How metaphysics ties in with the physics

Four examples:

1. The measurement rules of QM revisited
2. Limitations to knowledge
3. The natural configuration space of identical particles
4. Superselection rules
How metaphysics ties in with the physics (1): The measurement rule revisited

- Physics textbooks usually talk as if it were obvious which operator corresponds to which experimental setup. Can you provide a formula for the right operator?
- Usually, one says that observables = self-adjoint operators, and possible outcomes correspond to projections.
- But actually, in general, possible outcomes correspond to operators $0 \leq E \leq I$, and observables to POVMs (positive-operator-valued measures).
- The question how to collapse $\psi$ in this general case is somewhat involved and leads to “completely positive superoperators.”
- What if the run-time $T$ of the experiment is not determined in advance but chosen by the experiment (as when we are waiting for a detector to click)?
- What is the collapsed wave function at the random time $T$?
- What if the whole experiment is random? Starting at a random time, on a random system, using a random apparatus in a random state and a random interaction?
Moral #1:

All these points can be analyzed in theories that treat quantum measurements not by axioms but allow their analysis (such as Bohm’s and GRW’s).


Moral #2:

If you wanted to formulate the measurement rule in a careful and comprehensive way, it would become excessively complicated. An ontological formulation (such as Bohm’s and GRW’s) of QM is way simpler.
A limitation to knowledge means that there is a fact whether the world has property $P$ or not but there is no empirical way of finding out.

A widespread kind of positivism:

Only operational statements ("if you do experiment X then the result is Y with probability P") are meaningful or scientific, only measurable quantities are well defined, and only observable objects are real.

That’s a bad view, quite exaggerated! The truth is much more complicated than that. See also [Cowan, Tumulka arXiv:1307.0827]

John Bell in 1987:

“To admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics.”
Theorem in Bohmian mechanics, and a “theorem” in ordinary QM

You cannot measure a particle’s wave function: There is no experiment that could be applied to any given particle with unknown wave function $\psi$ and would determine $\psi$.

If you prepare a particle in a spin state $\psi \in \mathcal{H} = \mathbb{C}^2$, then any experiment I can do yields just 1 bit of probabilistic information about $\psi$.

If I am given $N \gg 1$ particles, each with wave fct $\psi$, then I can determine $\psi$ to arbitrary accuracy and reliability if $N$ is sufficiently large.

If you know that a certain particle has wave fct $\psi$ then you can prove it, in the following sense: You can specify an experiment (with observable $P_{\psi}$) that yields outcome “1” with prob. 1 and “0” with prob. 0; if you didn’t know $\psi$ the prob. of “0” would be positive.

Upshot: Nature can keep a secret. She knows what the wave function is, but doesn’t allow us to measure it. Quantum mechanics proves positivism wrong.
GRWf and GRWm make exactly the same predictions. So if we live in a GRWf or GRWm world, we cannot tell which one.

Some people don’t like Bohmian mechanics because you can’t observe the entire trajectory without disturbing it. Not a good reason.
For $N$ identical particles in Bohmian mechanics, one simply needs to assume the same symmetrization postulate as in standard QM.

**Symmetrization Postulate**

If particles $i$ and $j$ belong to the same species then either

$$
\psi(\ldots x_i \ldots x_j \ldots) = \psi(\ldots x_j \ldots x_i \ldots) \quad \text{(bosons)}
$$

or

$$
\psi(\ldots x_i \ldots x_j \ldots) = -\psi(\ldots x_j \ldots x_i \ldots) \quad \text{(fermions)}
$$

But if we ask *why* that is so, then more can be said:
The configuration space of $N$ identical particles

[Laidlaw, De Witt 1971; Leinaas, Myrheim 1977]

Space of unordered configurations $q = \{x_1, \ldots, x_N\}$,

$$N\mathbb{R}^3 = \{q \subset \mathbb{R}^3 : \#q = N\},$$

as opposed to the space of ordered configurations $\hat{q} = (x_1, \ldots, x_N)$,

$$(\mathbb{R}^3)^N.$$

$N\mathbb{R}^3$ is a topologically non-trivial manifold.

$$N\mathbb{R}^3 = \left( (\mathbb{R}^3)^N \setminus \{\text{collisions}\} \right) / \text{permutations}$$

where $\{\text{collisions}\} = \{(x_1, \ldots, x_N) : \exists i \neq j \text{ with } x_i = x_j\}$. 

Another topologically non-trivial manifold.
In Bohmian mechanics, where the word “particle” is taken literally, it seems *clear-cut* that $N\mathbb{R}^3$ is the natural configuration space of $N$ identical particles in $\mathbb{R}^3$. (In standard QM, this is less compelling.)

On any topologically non-trivial manifold $Q$ in the place of the configuration space, there are several Schrödinger equations corresponding to different choices of topological factors $\gamma$ (unitary representations of the first homotopy group $\pi_1(Q)$). These theories use the “unfolding” $\hat{Q}$ of $Q$ (universal covering space).

In Bohmian mechanics, these Schrödinger equations arise naturally, with $\psi: \hat{Q} \to \mathbb{C}$ and $Q \in Q$. (In standard QM, they are less compelling.)

For $Q = N\mathbb{R}^3$, $\pi_1(Q)$ is the permutation group, and there exist two topological factors,

$\gamma = 1$ corresponding to bosons and

$\gamma = \text{sign(permation)}$ corresponding to fermions.
A superselection rule says that

in a particular model/situation, not every self-adjoint operator is an observable—\(A\) is an observable if and only if \(AS = SA\) for a certain ("superselected") operator \(S\) (usually highly degenerate). Equivalently, it says that it is impossible to distinguish empirically between a superposition \(\psi = \sum_i c_i \phi_i\) of eigenvectors \(\phi_i\) of \(S\) and the mixture \(\{\phi_i\text{ with weight } |c_i|^2\}\).

How could you **prove** a superselection rule if, as in orthodox QM, measurements are unanalyzable and observables are introduced by axiom? But you **can** in Bohmian mechanics and GRW theory!


A clear ontology allows for clear statements and clear proofs.
Thank you for your attention