Given formulas: The general solution of the first order linear ODE \( y' + p(t)y = g(t) \) is
\[
y(t) = \frac{1}{\mu(t)} \left( \int \mu(t) g(t) \, dt + c \right), \quad \mu(t) = e^{\int p(t) \, dt}.
\]
A particular solution of \( y'' + p(t)y' + q(t)y = g(t) \) is (variation of parameters)
\[
Y(t) = -y_1(t) \int_{t_0}^{t} \frac{y_2(s)g(s)}{W(y_1, y_2)(s)} \, ds + y_2(t) \int_{t_0}^{t} \frac{y_1(s)g(s)}{W(y_1, y_2)(s)} \, ds,
\]
where \( \{y_1, y_2\} \) is a fundamental set of solutions of \( y'' + p(t)y' + q(t)y = 0 \).

Trigonometric identities:
\[
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta, \quad \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta.
\]

Time: 3 hours. Calculators, books and notes are not allowed. Good luck!
6) Find the general solution of
\[ \frac{dx}{dt} = x - y, \quad \frac{dy}{dt} = -x + 3y. \]

7) Consider the system
\[ \frac{dx}{dt} = -2x - 2y - xy - y^2, \quad \frac{dy}{dt} = -y + xy. \]
Find all critical points, classify each as to type (saddle point, etc.), and state for each one whether it is unstable, stable, or asymptotically stable.

8) (a) Show that the differential equation
\[ (2x^2y - 3x^2 - 4y)\frac{dy}{dx} + (2xy^2 - 6xy) = 0 \]
is exact.

(b) Find an equation of the form \( H(x,y) = c \) satisfied by the trajectories \( y(x) \) of the system
\[ \frac{dx}{dt} = 2x^2y - 3x^2 - 4y, \quad \frac{dy}{dt} = -2xy^2 + 6xy. \]

9) Find the general solution of \( t^2y'' - 2y = 3t^2 - 1, \ t > 0. \) Instructions: Use the power ansatz \( y(t) = t^r \) to obtain a fundamental set of solutions of the corresponding homogeneous equation; then use variation of parameters to obtain a particular solution.

10) Find the general solution of \( y''' - y'' - y' + y = e^t + 1. \)

11) [10 points] A home buyer can afford to spend no more than $1000/month on mortgage payments. Assume that interest is compounded continuously and that payments are also made continuously. Suppose the interest rate is \( r = 0.05/yr \) and that the term of the mortgage is 20 years.

(a) Set up a DE for the amount of debt \( y(t) \) after \( t \) years.

(b) Determine the maximum amount \( M \) that this buyer can afford to borrow. (It is sufficient to obtain an expression that could be easily evaluated with a calculator.)