

Corrections to “Intro. to Homological Algebra” by C. Weibel

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- p.2 line -12: d_{n-1} should be d_n
- p.4 lines 5,6: $V - E - 1$ should be $E - V + 1$ (twice)
- p.4 lines 7,8: all 5 occurrences of v_0 should be replaced by v_1 .
- p.6, line 7 of Def. 1.2.1: “non-abelian” should be “non-additive”
- p.8 diagram: the upper right entry should be $C_{p+1,q+1}$, not $C_{p+1,p+1}$.
- p.12 line 1: $B \rightarrow C$ should be $B \xrightarrow{p} C$
- p.12 line 9: “so is $\text{coker}(f) \rightarrow \text{coker}(g)$ ” should be “so is $\text{coker}(g) \rightarrow \text{coker}(h)$ ”
- p.13 line -1: $Z_{n-1}(b)$ should be $Z_{n-1}(B)$
- p.15 line 11: $B(-1)$ should be $B[-1]$
- p.18 line 3: Replace the sentence “Give an example...” with: “Conversely, if C and $H_*(C)$ are chain homotopy equivalent, show that C is split.”
- p.18 line 18: Replace $i = 1, 2$ with $i = 0, 1$
- p.19 line -7: ‘split complexes’ should be ‘exact complexes’
- p.21 Ex.1.5.3: Add extra paragraph: If $f : B \rightarrow C$, $g : C \rightarrow D$ and $e : B \rightarrow C$ are chain maps, show that e and gf are chain homotopic if and only if there is a chain map $\gamma = (e, s, g)$ from $\text{cyl}(f)$ to D . Note that e and g factor through γ .
- p.24 line -7: ∂ should be α
- p.26 line -10: { let } $C^\infty(U)$ be ... that C^∞ is a sheaf...
- p.27 line 7: the contour integral should be $\frac{1}{2\pi i} \oint f'(z)dz/f(z)$, not $\frac{1}{2\pi i} \oint f(z)dz$.
- p.29 line 17: should read “[Freyd, p. 106], every small full abelian subcategory of \mathcal{L} is equivalent to a full abelian subcategory of the category $R\text{-mod}$ of modules over the ring”
- p.32 line 1: 2.6.3 should be 2.6.4
- p.33 line -2: after “no projective objects” add “except 0.”
- p.34 line -14 (Ex. 2.2.1): Add this sentence before the hint: “Their brutal truncations $\sigma_{\geq 0}P$ form the projective objects in $\mathbf{Ch}_{\geq 0}$.”
- p.35 line 8: replace “chain map” by “quasi-isomorphism”
- p.37 line 1: delete ‘commutative’
- p.40 line -8: the map F should be f
- p.43 Ex. 2.3.8: $\mathcal{A}^{(I^{op})}$ should be $(\mathcal{A}^{op})^{(I^{op})}$
- p.44 line 11: ‘gf’ should be ‘ gf ’ (math font)
- p.44 line -9: $L_i(A)$ should be $L_iF(A)$
- p.45 line 11: ‘UF’ should ‘ UF ’
- p.47 line -6: the m^{th} syzygy
- p.49 line 1: $L_n(f)$ should be $L_nF(f)$
- p.49 line -11 (Ex. 2.4.4): Replace “the mapping cone $\text{cone}(A)$ of exercise 1.5.1” by the following text: “ $\sigma_{\geq 0}\text{cone}(A)[1]$, where $\text{cone}(A)$ is the mapping cone of exercise 1.5.1. If \mathcal{A} has enough projectives, you may also use the projective objects in $\mathbf{Ch}_{\geq 0}(\mathcal{A})$, which are described in Ex. 2.2.1.”
- p.50 line -10: $\text{Hom}_R(, B)$ -acyclic should be $\text{Hom}_R(A,)$ -acyclic.
- p.55 line -9 to -6: Replace paragraph with:
We say that \mathcal{A} satisfies axiom (AB4) if it is cocomplete and direct sums of monics are monic, i.e., homology commutes with direct sums. This is true for \mathbf{Ab} and $\mathbf{mod}\text{-}R$. (Homology does not commute with arbitrary colimits; the derived functors of colim intervene via a spectral sequence.) Here are two consequences of axiom (AB4).

- p.55 line -5: delete “cocomplete” and insert “satisfying (AB4)” before “has enough projectives”
- p.56 line 13: (1) and (2) should be switched
- p.57 lines 2,-10: a is the image (‘a’ should be ‘ a ’ twice)
- p.57 line 4: $a_{jk} \in A_j$ should be $a_j \in A_j$
- p.58 lines 6–7: Replace the text “If...and” with: Suppose that $\mathcal{A} = R\text{-mod}$ and $\mathcal{B} = \mathbf{Ab}$ (or \mathcal{A} is any abelian category with enough projectives, and \mathcal{A} and \mathcal{B} satisfy axiom (AB5)). If”
- p.58 line 9: $F(A)$ should be $F(A_i)$
- p.60–61: several 2-symbol subscripts are missing the comma (e.g., C_{pq} means $C_{p,q}$).
- p.62 lines 8: Replace the sentence “Finally...acyclic.” with: “Show that $\text{Tot}^\oplus(D)$ is not acyclic either.”
- p.66 line 9: $pB = 0$ should be $pb = 0$.
- p.67 line 5: Tor_* should be Tor_1
- p.70 end of line -11: $j =$ should be $i =$
- p.71 Proof of 3.4.1: ... applying $\text{Ext}^*(-, B)$ yields the exact sequence

$$\text{Hom}(X, B) \rightarrow \text{Hom}(B, B) \xrightarrow{\partial} \text{Ext}^1(A, B)$$

- so the identity map id_B lifts to a map $\sigma : X \rightarrow B$ when $\text{Ext}^1(A, B) = 0$. As σ is a section of $B \rightarrow X$, ...
- p.71 lines 7–8: ... the class $\Theta(\xi) = \partial(\text{id}_B) \dots \text{id}_B$ lifts to $\text{Hom}(X, B)$ iff ...
- p.73 line 7: Tor_m should be Tor_n
- p.74 Exercise 3.3.1: $\dots \cong \mathbb{Z}_{p^\infty}$ should be $\dots \cong (\mathbb{Q}/\mathbb{Z}[1/p]) \times \hat{\mathbb{Q}}_p/\mathbb{Q}$.
- p.74 Exercise 3.3.5: In the display, replace A/pA with A^*/pA^* and delete the final ‘ $= 0$ ’. On the next line (line -1), ‘ A is divisible’ should be ‘ A^* is divisible, i.e., A is torsionfree’.
- p.79 line -6: “ X_n and X_n'' under B , and let Y_n be the...copy of B .” should be “ X_n and X_n' under B .”
- p.79 line -4 (display): Y_n should be X_n''
- p.82 line 6: ... axiom (AB5*) (filtered limits are exact), the above proof can be modified to show ...
- p.82 line 9: add sentence: Neeman has given examples of abelian categories with (AB4*) in which Lemma 3.5.3 and Corollary 3.5.4 both fail; see *Invent. Math.* 148 (2002), 397–420.
- p.82 line -8: ‘complete’ should be ‘complete and Hausdorff’
- p.85 line -4: Then $\text{Tot}(C) = \text{Tot}^\Pi(C)$ is ...
- p.86 line 2: “nonzero columns.” (not rows)
- p.89 line 8: $\} \otimes \{$ should be $\} \oplus \{$.
- p.90 line 8: $\prod_{p+q} n_{-1}$ should be $\prod_{p+q=n-1}$
- p.93 line -3: ‘all R -modules B ’ should be ‘all R -modules A ’
- p.95 line 17: ‘the’ (before $pd_R(P)$) should be ‘then’
- p.96 line -11: replace ‘integer m ’ with ‘ $m \neq 0$ ’
- p.97 line 14: Add sentence:
 “If in addition R is finite-dimensional over a field then R is quasi-Frobenius $\Leftrightarrow R$ is Frobenius.”
- p.101 line -5: $n < \infty$ should be $d < \infty$
- p.102 lines 3,4: $\leq 1 + n$ should be $\leq 1 + d$ twice
- p.107 line -8: after $G(R) \leq \text{id}(R)$ insert ‘, and $\text{id}(R) = \dim(R)$ by 4.2.7’
- p.113 line 13: the final $H_q(C)$ should be $H_{q-1}(C)$
- p.122 line 9: $-(r+1)/r$ should be $-(r-1)/r$
- p.124 line 7: E_{0n}^∞ is a quotient of E_{0n}^a and each E_{n0}^∞ is a subobject of E_{n0}^a .
- p.124 line -7: $0 \rightarrow E_{0n}^2 \rightarrow H_n \rightarrow E_{1,n-1}^2 \rightarrow 0$.
- p.127 line 14 (**): $(-1)^{p_1}$ should be $(-1)^{p_1+q_1}$

p.127 line 16: replace “and (**) for every $r \geq a$. We shall” by “for every $r \geq a$. If the induced product on E^r satisfies (**) for all $r \geq a$, we shall”

p.131 lines 3–4: $SO(1)$ should be $SO(2)$ twice

p.131 line 7: Replace “ $H_2(SO(3)) \cong \mathbb{Z}$, ...isomorphism.” with “ d^2 is an injection, and $H_2(SO(3)) = 0$.”

p.132 line -3: $F_s H_n(C)$ should be $F_s C_n$

p.134 line 8: The filtration on the complex C' is bounded below, the one on C'' ...

p.135 line 5: although correct as stated, it would be more clear if it read $E_{pq}^r(F) \cong E_{2p+q,-p}^{r+1}(\tilde{F})$ and $E_{pq}^r(F) \cong E_{-q,p+2q}^{r-1}(\text{Dec}F)$ so that the substitution $n = p + q$ is not needed.

p.135 line 12: the superscripts r should be $r + 1$, viz.,

$$\cdots E_{p+r}^{r+1}(\text{cone } f) \rightarrow E_p^{r+1}(B) \rightarrow E_p^{r+1}(C) \rightarrow E_p^{r+1}(\text{cone } f) \rightarrow E_{p-r}^{r+1}(B) \cdots$$

p.135 line 18: Insert after $d(c) = 0$: “(This assumes that (AB5) holds in \mathcal{A} .)”

p.135 line -13: after 'exhaustive' add: “and that \mathcal{A} satisfies (AB5).”

p.135 line -4: replace text starting with E_{p0}^1 to read: E_{p0}^1 is $\bar{C}_p = C_p / (F_{p-1}C_p + d(F_p C_{p+1}))$; \bar{C} is the top quotient chain complex of C , and $d_{p0}^1 : E_{p0}^1 \rightarrow E_{p-1,0}^1$ is induced from $d : C_p \rightarrow C_{p-1}$.

p.136 line -9,-8: let $F_{-p}C$ be $2^p C$ ($p \geq 0$).

p.136 line -7: “Each row” should be “Each column”

p.137 Cor. 5.5.6 should read: If the spectral sequence weakly converges, then the filtrations on $H_*(C)$ and $H_*(\hat{C})$ have the same completions.

p.143 line 15: $H_q(A)$ should be $H_q(Q)$

p.145 line -10: insert 'with $\epsilon d^v = 0$ before 'such that'

p.152 line 8: $\xrightarrow{\otimes_S R}$ should read $\xrightarrow{\otimes_R S}$.

p.154 line -2: $\mathcal{E} = \mathcal{E}^{a+1}$ and \mathcal{E}^r denotes the $(r - a - 1)^{st}$ derived couple

p.154 line -1: $j^{(r)}$ has bidegree $(1 - r, r - 1)$

p.155 line 2: remove $\xrightarrow{i} D$ from diagram to read: $E_{pq}^r \xrightarrow{k} D_{p-1,q}^r \xrightarrow{j^{(r)}} E_{p-r,q+r-1}^r$.

p.155 line 6: starting with E^{a+1} .

p.160 display on line -7: delete ' and a in A '

p.163 line -12: (3.2.29) should be (3.2.9)

p.168 line -1: $H_{1-n}(G; A)$ should be $H_{-1-n}(G; A)$

p.173 line -5: $(\sigma - 1)K$ should be $(\sigma - 1)L$

p.177 line 13: If m is odd, every automorphism of D_m stabilizing C_m is inner.

p.185 fourth line of proof of Classification Theorem: $\beta(1)$ should be $\beta(1) = 1$

p.186 line -3: $b_h g$ should be $b_h h$.

p.184 lines 11–12: $(1, g)$ should be $(0, g)$, and $(1, h)$ should be $(0, h)$

p.191 Cor. 6.7.9: ... of G on H induces an action of G/H on $H_*(H; \mathbb{Z})$ and $H^*(H; \mathbb{Z})$.

p.191 line -3: complex of (space missing)

p.193 line 19: delete ' $\beta\sigma = 0$ ' so it reads ' $(\sigma^2 = 0)$ '

p.193 lines -3, -8; and p.194 line 7: “cocommutative” should be “coassociative”

p.196 line -4: If H is in the center of G and A is a trivial G -module then G/H acts trivially ...

p.197 Example 6.8.5: The two occurrences of D_{2m} should read D_m (on the first and last lines).

p.201 line 11: $f = f'$ should be $f_1 = f_2$

p.201 Exercise 6.9.2: If ... and ... are central extensions, and X is perfect, show ...

p.203 lines 1–2: When \mathbb{F}_q is a finite field, and $(n, q) \neq (2, 2), (2, 3), (2, 4), (2, 9), (3, 2), (3, 4), (4, 2)$, we know that $H_2(SL_n(\mathbb{F}_q); \mathbb{Z}) = 0$ [Suz, 2.9]. With these exceptions, it follows that

p.206 line 8: $H_q(S_n(X) \otimes_{\mathbb{Z}} A)$ should be $H_q(G; S_n(X) \otimes_{\mathbb{Z}} A)$

p.213 Exercise 6.11.11: ... Show that for $i \neq 0$:

$$H^i(G; \mathbb{Z}) = \begin{cases} \mathbb{Z}_{p^\infty} & i = 2 \\ 0 & \text{else.} \end{cases}$$

p.226: Line 3 of Exercise 7.3.5 should read: δ -functors (assuming that that k is a field, or that N is a projective k -module):

p.238 lines 4–7: Replace these two sentences (Show that...it suffices to show that ...= 0.) by:

Conversely, suppose that $\mathfrak{g} = \mathfrak{f}/\mathfrak{r}$ for some free Lie algebra \mathfrak{f} with $\mathfrak{r} \subseteq [\mathfrak{f}, \mathfrak{f}]$, and \mathfrak{g} is free as a k -module. Show that if $H^2(\mathfrak{g}, M) = 0$ for all \mathfrak{g} -modules M then \mathfrak{g} is a free Lie algebra. *Hint*: It suffices to show that ...= 0.

p.255 line -8: $0 \leq i_s \leq \dots \leq i_1 \leq m$ should be $0 \leq i_s < \dots < i_1 \leq m$

p.256 line 8: identity (not identify)

p.257 line 15 (display): $\alpha_*(t)$ should be $\alpha_*(s)$

p.258 lines 1, 20 and -7: ‘combinational’ should be ‘combinatorial’

p.261 lines -9,-8,-6: ‘combinatorial’ is misspelled three more times

p.262 line 18: $g_r = g(\sigma_r u)^{-1}$

p.262 line -13: ‘every n ’ should be ‘every sufficiently large n ’

p.262 line -6: $\partial_i(y)$ should be $\partial_i(y) = x_i$

p.264 lines 2–3: the two occurrences of $S(X)$ should read $S(|X|)$

p.265 add to end of Exercise 8.3.3:

Extend exercise 8.2.5 to show that a homomorphism of simplicial groups $G \rightarrow G''$ is a Kan fibration if and only if the induced maps $N_n G \rightarrow N_n G''$ are onto for all $n > 0$. In this case there is also a long exact sequence, ending in $\pi_0(G'')$.

p.266 line 14: that $\sigma_i(x_i) \neq 0$, then $y = y - \sigma_i \partial_i y = \sum_{j>i} \sigma_j(x'_j)$. By induction, $y = 0$. Hence $D_n \cap N_n = 0$.

p.267 line 5: fix subscript on sum: $d\sigma_p(x) = \sum_{p+2}^n$

p.267 line 6: $d\sigma_p^2(x) + \sigma_p d\sigma_p(x) = \sum_{i=p+2}^{n+1} \dots + \sum_{i=p+2}^n$

p.267 line 7: $= (-1)^p \sigma_p(x)$.

p.267 line 8: Hence $\{s_n = (-1)^p \sigma^p\}$

p.268 line -3: ‘ $\{0, 1, \dots, i-1\}$ ’ should be ‘ $\{0, 1, \dots, i\}$ ’

p.273: the middle display should read

$$\text{Hom}_{\mathbf{Ch}}(NA, C) \cong \text{Hom}_{\mathcal{S}\mathcal{A}}(A, K(C)).$$

p.278 display on line 8: 1 should be subtracted from the subscripts: $\sigma_{\mu(n)-1}^h \cdots \sigma_{\mu(p+1)-1}^h \sigma_{\mu(p)-1}^v \cdots \sigma_{\mu(1)-1}^v$

p.280 lines 10–11: $\eta: 1_{\mathcal{C}} \rightarrow UF$ and ... $\varepsilon: FU \rightarrow 1_{\mathcal{B}}$. (switch \mathcal{B} and \mathcal{C})

p.283 line -2: $\bar{r}_i \bar{r}_{i+1}$ should be $\overline{\bar{r}_i \bar{r}_{i+1}}$

p.287 lines -5, -4: “is an exact sequence” should be “is a sequence” and “is also exact” should be “is exact”

p.290 line before 8.7.9: Insert sentence: The proof of Theorem 3.4.3 goes through to prove that $\text{Ext}_{R/k}^1(M, N)$ classifies equivalence classes of k -split extensions of M by N .

p.291 line -2: ... to $(R/I)^d$. If each $x_i R \subset R$ is k -split then:

p.294 line -8: If M is an R -module, (‘a k ’ should be ‘an R ’)

p.295 line -4 (display): $D^1(R, M)$ should be $D^1(R/k, M)$

p.296 8.8.6: “If k is a field” should be “If R is a field”

p.297, line -9: the sequence should read

$$\cdots \rightarrow D_{n+1}(R/K, M) \rightarrow D_n(K/k, M) \rightarrow D_n(R/k, M) \rightarrow D_n(R/K, M) \rightarrow D_{n-1}(K/k, M) \rightarrow \cdots$$

p.298 line 2 of 8.8.7: *Commalg* should be in roman font: **Commalg**

p.301 lines 4–5: the ranges should be “if $0 < i \leq n$ ” and “if $i = n + 1$ ” respectively.

p.304 Exercise 9.1.3: the variable n should m each time ($y_n, R^n, n, p < n$); and x (on line 17) should be \mathbf{x}

p.307 line -1 should read:

As $\mathrm{Tor}_1^{R^e/k}(R^e, M) = 0$, the long exact relative Tor sequence (Lemma 8.7.8) yields

p.322: On line 1, insert “If $1/2 \in k$,” before “ $\Omega_{R/k}^*$ is the free graded-commutative” and (on line 4) add the sentence: In general, $\Omega_{R/k}^*$ is the free alternating R -algebra generated by $\Omega_{R/k}^1$.

p.325 line -6: $(-1)^n$ should be $(-1)^\sigma$

p.354 line -6: Add sentence: It also follows from the Connes-Karoubi theorem on noncommutative de Rham homology in C.R. Acad. Sci. Paris, t. 297 (1983), p. 381–384.

p.353 line 7: $u = c\epsilon_n$ ” should read ”... $u = c\epsilon_{n+1}$ ”

p.359 Exercise 9.9.5: This is wrong; replace it with:

Exercise 9.9.5 (Grauert-Kerner) Consider the artinian algebra $R = k[x, y]/(\partial f/\partial x, \partial f/\partial y, x^5)$, where $f = x^4 + x^2y^3 + y^5$. Show that $I = (x, y)R$ is nilpotent, and f is a nonzero element of $H_{dR}^0(R)$ which vanishes in $H_{dR}^0(R/I)$.

p.370 line -6: [ho-]“mopy” should be [ho-]“motopy” and $b, ''$ should be b'' ,

p.375 line 4: $u\delta = ig : B' \rightarrow B$ should be $T(u)\delta = ig : B' \rightarrow T(B)$

p.376 line -10: diagram ... commutes up to chain homotopy.

p.381 line 6: ‘(left)’ should be ‘(right)’

p.382 lines-6,-7 (10.3.8): the two occurrences of ‘right’ should be ‘left’

p.383 line 7 (Corollary 10.3.10): after ‘object’ add ‘, and that S is saturated.’ Add to the end of the proof the sentence: “So S contains maps $Y \xrightarrow{0} X \xrightarrow{0} Z$, and hence $X \xrightarrow{0} X$.”

p.384 lines 9, 11: ‘left’ fraction should be ‘right’ fraction

p.384 line 13: Replacing ‘right’ by ‘left’

p.385 line 1: \mathcal{B} is a small category and $\mathrm{Ext}(A, B)$ is a set for all $A \in \mathcal{A}, B \in \mathcal{B}$. Then show that...

p.386 lines 7–9: six occurrences of g should be v : ...there should be a $v : X \rightarrow Z$... $f - g = uv$. Embed v in an exact triangle (t, v, w) ... Since $vt = 0$, $(f - g)t = uvt = 0$, ...

p.386 lines 14–18: Replace the two sentences “Given $us_1^{-1} : \dots$ triangle in \mathbf{K} ” with: “The exact triangles in $S^{-1}\mathbf{K}$ are defined to be those triangles which are isomorphic, in the sense of (TR1), to the image under $\mathbf{K} \rightarrow S^{-1}\mathbf{K}$ of an exact triangle in \mathbf{K} .”

p.386 line 20: replace “straightforward but lengthy; one uses the fact” with “straightforward; one uses (TR3) and the fact that...”

p.387 line -12 (10.4.5): delete ‘well-powered’ (Gabber points out that this condition is superfluous).

p.388 lines 2–3: the ‘ $-dky$ ’ and ‘ $+dk$ ’ should be ‘ $+dky$ ’ and ‘ $-dk$ ’

p.396 line 10: [Hart, II.5]

p.396 line 12: [Hart, exercise III.6.4] should be [HartRD, II.1.2]

p.400 line 8: the last two (A, B) should be $(A, T^n B)$

p.400 line -7: $\mathrm{Hom}(-, B)$ should be $\mathrm{Hom}^\cdot(-, B)$

p.405 line -2: a natural homomorphism in $\mathbf{D}(R)$, which is an isomorphism if either each C_i is fin. gen. projective or else A is quasi-isomorphic to a bounded below chain complex of fin. gen. projective R -modules:

p.420 line 13: in exercise 6.11.3 (not 6.11.4)

p.427 line -1: $I \in I$ should be $i \in I$

p.428 line 16: $F_i \rightarrow F_i \rightarrow C$ should be $F_j \rightarrow F_i \rightarrow C$

p.429 line 15: 'Chapter 1' should read 'Chapter 2'

p.431 line 6: 'functions' should be 'functors.'

p.435 under 'AB4 axiom': add page 55

p.439 add entry to Index under 'double chain complex':

Connes' — \mathcal{B} . See Connes' double complex.

p.444, under 'Lie group': [page] 158 should be 159

p.445, line -6: 'Øre' should be 'Ore'

p.448 column 2: lines 29-30 should only be singly indented ("— of" refers to spectral sequence)

References

[EM] Eilenberg, S., and Moore, J. "Limits and Spectral Sequences." *Topology* **1** (1961): 1–23.