Corrections to “Intro. to Homological Algebra” by C. Weibel
Cambridge University Press, paperback version, 1995

p.2 line -12: $d_{n-1}$ should be $d_n$
p.4 lines 5,6: $V - E - 1$ should be $E - V + 1$ (twice)
p.4 lines 7,8: all 5 occurrences of $v_0$ should be replaced by $v_1$.
p.6, line 7 of Def. 1.2.1: “non-abelian” should be “non-additive”
p.8 diagram: the upper right entry should be $C_{p+1,q+1}$, not $C_{p+1,p+1}$.
p.12 line 1: $B \to C$ should be $B \to C$
p.12 line 9: “so is $\text{coker}(f) \to \text{coker}(g)$” should be “so is $\text{coker}(g) \to \text{coker}(h)$”
p.13 line -1: $Z_{n-1}(b)$ should be $Z_{n-1}(B)$
p.15 line 11: $B(-1)$ should be $B[-1]$
p.18 line 3: Replace the sentence “Give an example...” with: “Conversely, if $C$ and $H_*(C)$ are chain homotopy equivalent, show that $C$ is split.”
p.18 line 18: Replace $i=1,2$ with $i=0,1$
p.21 Ex.1.5.3: Add extra paragraph: If $f : B \to C$, $g : C \to D$ and $e : B \to C$ are chain maps, show that $e$ and $gf$ are chain homotopic if and only if there is a chain map $\gamma = (e, s, g)$ from $\text{cyl}(f)$ to $D$. Note that $e$ and $g$ factor through $\gamma$.
p.24 line -7: $\partial$ should be $\alpha$
p.26 line -10: { let } $C^\infty(U)$ be ... that $C^\infty$ is a sheaf...
p.27 line 7: the contour integral should be $\frac{1}{2\pi i} \oint f'(z)dz/f(z)$, not $\frac{1}{2\pi i} \oint f(z)dz$.
p.29 line 17: should read “[Freyd, p. 106], every small full abelian subcategory of $L$ is equivalent to a full abelian subcategory of the category $R\text{-mod}$ of modules over the ring”
p.32 line 1: 2.6.3 should be 2.6.4
p.34 line -14 (Ex. 2.2.1): Add this sentence before the hint: “Their brutal truncations $\sigma_{\geq 0}P$ form the projective objects in $\text{Ch}_{\geq 0}.$”
p.35 line 8: replace “chain map” by “quasi-isomorphism”
p.37 line 1: delete ‘commutative’
p.40 line -8: the map $F$ should be $f$
p.43 Ex. 2.3.8: $A^{(rp)}$ should be $(A^{op})^{(rp)}$
p.44 line 11: ‘gf’ should be ‘gf’ (math font)
p.44 line -9: $L_i(A)$ should be $L_iF(A)$
p.45 line 11: ‘UF’ should ‘UF’
p.47 line -6: the $m^{th}$ syzygy
p.49 line 1: $L_n(f)$ should be $L_nF(f)$
p.49 line -11 (Ex. 2.4.4): Replace “the mapping cone $\text{cone}(A)$ of exercise 1.5.1” by the following text: “$\sigma_{\geq 0}\text{cone}(A)[1]$, where $\text{cone}(A)$ is the mapping cone of exercise 1.5.1. If $A$ has enough projectives, you may also use the projective objects in $\text{Ch}_{\geq 0}(A)$, which are described in Ex. 2.2.1.”
p.50 line -10: $\text{Hom}_R(\ , B)$-acyclic should be $\text{Hom}_R(A, \ )$-acyclic.
p.55 line -9 to -6: Replace paragraph with:
We say that $A$ satisfies axiom (AB4) if it is cocomplete and direct sums of monics are monic, i.e., homology commutes with direct sums. This is true for $\text{Ab}$ and $\text{mod-}R$. (Homology does not commute with arbitrary colimits; the derived functors of colim intervene via a spectral sequence.) Here are two consequences of axiom (AB4).
p.55 line -5: delete “cocomplete” and insert “satisfying (AB4)” before “has enough projectives”
p.56 line 13: (1) and (2) should be switched
p.57 lines 2–10: a is the image (‘a’ should be ‘a’ twice)
p.57 line 4: \(a_{jk} \in A_j\) should be \(a_j \in A_j\)
p.58 lines 6–7: Replace the text “If...and” with: Suppose that \(A = R\text{-mod}\) and \(B = Ab\) (or \(A\) is any abelian category with enough projectives, and \(A\) and \(B\) satisfy axiom (AB5)). If
p.58 line 9: \(F(A)\) should be \(F(A_i)\)
pp.60–61: several 2-symbol subscripts are missing the comma (e.g., \(C_{pq}\) means \(C_{p,q}\)).
p.62 lines 8: Replace the sentence “Finally...acyclic.” with: “Show that \(\text{Tot}^\oplus(D)\) is not acyclic either.”
p.66 line 9: \(pB = 0\) should be \(pb = 0\).
p.70 end of line -11: \(i = j\) should be \(i =\)
p.71 Proof of 3.4.1: ... applying \(\text{Ext}^*(-, B)\) yields the exact sequence

\[\text{Hom}(X, B) \to \text{Hom}(B, B) \xrightarrow{\varphi} \text{Ext}^1(A, B)\]

so the identity map \(id_B\) lifts to a map \(\sigma : X \to B\) when \(\text{Ext}^1(A, B) = 0\). As \(\sigma\) is a section of \(B \to X\), ...
p.71 lines 7–8: ... the class \(\Theta(\xi) = \partial(id_B) \ldots id_B\) lifts to \(\text{Hom}(X, B)\) iff ...
p.73 line 7: \(\text{Tor}_n\) should be \(\text{Tor}_n\)
p.74 Exercise 3.3.1: \(\cdots \cong \mathbb{Z}_{p^\infty}\) should be \(\cdots \cong (\mathbb{Q}/\mathbb{Z}[1/p]) \times \mathbb{Q}_p/\mathbb{Q}\).
p.74 Exercise 3.3.5: In the display, replace \(A/pA\) with \(A^*/pA^*\) and delete the final ‘\(= 0\)’. On the next line (line -1), ‘\(A\) is divisible’ should be ‘\(A^*\) is divisible, i.e., \(A\) is torsionfree’.

p.74 line 2: \(\text{Ext}^*(A, -)\) should be \(\text{Ext}^*(-, B)\)
p.74 line 3: \(\text{Hom}(X, B) \to \text{Hom}(B, B)\)
p.74 line 4: \(\sigma : X \to B\)
p.74 line 5: section of \(B \to X\)
p.74 lines 7–8: \(\Theta(\xi) = \partial(id_B) \ldots \text{id}_B\) lifts to \(\text{Hom}(X, B)\)
p.74 line 11: \(\Theta : \xi \to \partial(id_B)\)
p.79 line -6: “\(X_n\) and \(X'_n\) under \(B\), and let \(Y_n\) be the...copy of \(B\)” should be “\(X_n\) and \(X'_n\) under \(B\),”
p.79 line -4 (display): \(Y_n\) should be \(X'_n\)
p.82 line 6: ... axiom (AB5*) (filtered limits are exact), the above proof can be modified to show ...
p.82 line 9: add sentence: Neeman has given examples of abelian categories with (AB4*) in which Lemma 3.5.3 and Corollary 3.5.4 both fail; see \textit{Invent. Math.} 148 (2002), 397–420.
p.82 line -8: ‘complete’ should be ‘complete and Hausdorff’
p.84 line -8: 1960 is correct; the paper was published in 1962.
p.85 line -4: Then \(\text{Tot}(C) = \text{Tot}^\Pi(C)\) is ...
p.86 line 2: “nonzero columns.” (not rows)
p.89 line 8: \(\{\} \otimes \{\text{should be}\} \otimes \{\).
p.90 line 8: \(\prod_{p+q = n-1}^{p+q} \text{should be}\) \(\prod_{p+q = n-1}^{p+q}\)
p.93 line -3: ‘all \(R\)-modules \(B\)’ should be ‘all \(R\)-modules \(A\)
p.95 line 17: ‘the’ (before \(pd_R(P)\)) should be ‘then’
p.96 line -11: replace ‘integer \(m\)’ with ‘\(m \neq 0\)’
p.97 line 14: Add sentence:

“If in addition \(R\) is finite-dimensional over a field then \(R\) is quasi-Frobenius \(\Leftrightarrow R\) is Frobenius.”

2
p.101 line -5: \( n < \infty \) should be \( d < \infty \)

p.102 lines 3, 4: \( \leq 1 + n \) should be \( \leq 1 + d \) twice

p.107 line -8: after \( G(R) \leq \text{id}(R) \) insert ‘, and \( \text{id}(R) = \dim(R) \) by 4.2.7’

p.113 line 13: the final \( H_{q}(C) \) should be \( H_{q-1}(C) \)

p.122 line 9: \( -(r + 1)/r \) should be \( -(r - 1)/r \)

p.124 line 7: \( E_{0}^{r} \) is a quotient of \( E_{0}^{0} \) and each \( E_{n}^{0} \) is a subobject of \( E_{0}^{0} \).

p.124 line -7: \( 0 \to E_{0}^{2} \to H_{n} \to E_{1,n-1}^{2} \to 0 \).

p.127 line 14 (**): \( (1^{p}) \) should be \( (1)^{p^{1+q}} \)

p.127 line 16: replace “and (**) for every \( r \geq a \). We shall” by “for every \( r \geq a \). If the induced product on \( E' \) satisfies (**), then we shall”

p.131 lines 3–4: \( SO(1) \) should be \( SO(2) \) twice

p.131 line 7: Replace “\( H_{2}(SO(3)) \cong \mathbb{Z} \ldots \) isomorphism.” with “\( d^{2} \) is an injection, and \( H_{2}(SO(3)) = 0. \)”

p.132 line -3: \( F_{h}H_{q}(C) \) should be \( F_{h}C_{n} \)

p.134 line 8: The filtration on the complex \( C' \) is bounded below, the one on \( C'' \)...

p.135 line 5: although correct as stated, it would be more clear if it read \( E_{pq}^{r}(F) \cong E_{2p+q, -p}^{r+1}(\tilde{F}) \) and \( E_{pq}^{r}(F) \cong E_{-q, p+1}^{r-1}(\text{DecF}) \) so that the substitution \( n = p + q \) is not needed.

p.135 line 12: the superscripts \( r \) should be \( r + 1 \), viz.,

\[
\cdots E_{p}^{r+1}(\text{cone } f) \to E_{p}^{r+1}(B) \to E_{p}^{r+1}(C) \to E_{p}^{r+1}(f) \to E_{p-r}^{r+1}(B) \cdots .
\]

p.135 line 18: Insert after \( d(c) = 0 \): “(This assumes that (AB5) holds in \( A \)).”

p.135 line 13: after ‘exhaustive’ add: “and that \( A \) satisfies (AB5).”

p.135 line -4: replace text starting with \( E_{q}^{1} \) to read: \( E_{q}^{1,0} \) is \( \tilde{C}_{q, p} = C_{p}/(F_{q-1}C_{p} + d(F_{q}C_{p+1})) \); \( \tilde{C} \) is the top quotient chain complex of \( C \), and \( d_{q}^{1,0} : E_{q}^{1,0} \to E_{q}^{1,0} \) is induced from \( \tilde{d} : C_{p} \to C_{p-1} \).

p.136 lines 2–3: the sequence should read \( 0 \to F_{p-1}C \to C \to \tilde{C} \to 0 \)

p.136 line -9–8: let \( F_{p}C \) be \( 2pC \) (\( p \geq 0 \)).

p.136 line -7: “Each row” should be “Each column”

p.137 Cor. 5.5.6 should read: If the spectral sequence weakly converges, then the filtrations on \( H_{s}(C) \) and \( H_{s}(\tilde{C}) \) have the same completions.

p.142 line 16: insert ‘if it is regular’ before ‘, and we have’

p.143 line 15: \( H_{q}(A) \) should be \( H_{q}(\tilde{Q}) \)

p.145 line -10: insert ‘with \( d \in E' \) = 0 before ‘such that’

p.152 line 8: \( \otimes_{R}^{S} \) should read \( \otimes_{R}^{S} \).

p.154 line -2: \( E = E^{a+1} \) and \( E' \) denotes the \( (r - a - 1)^{st} \) derived couple

p.154 line -1: \( j^{(r)} \) has bidegree \( (1 - r, r - 1) \)

p.155 line 2: remove \( i \to D \) from diagram to read: \( E_{pq}^{r} \to D_{p,q}^{r} \to E_{p-r,q+r-1}^{r} \).

p.155 line 6: starting with \( E^{a+1} \).

p.158 line -7: \( \ell H_{n} + T_{n} \) should be \( \ell H_{n} + T_{n} \)

p.160 display on line -7: delete ‘ and \( a \) in \( A' \)

p.163 line 12: \( (3.2.29) \) should be \( (3.2.9) \)

p.168 line -7: \( NA = 0 \) should be \( Na = 0 \)

p.168 line -1: \( H_{-1,n}(G; A) \) should be \( H_{-1-n}(G; A) \)

p.173 line -5: \( (\sigma - 1)K \) should be \( (\sigma - 1)L \)

p.177 line 13: If \( m \) is odd, every automorphism of \( D_{m} \) stabilizing \( C_{m} \) is inner.
that if $H$ should be $b_hg$ should be $b_hh$.

p.184 lines 11–12: $(1, g)$ should be $(0, g)$, and $(1, h)$ should be $(0, h)$

p.191 Cor. 6.7.9: ... of $G$ on $H$ induces an action of $G/H$ on $H_*(H; \mathbb{Z})$ and $H^*(H; \mathbb{Z})$.

p.191 line -3: complex of (space missing)

p.193 line 19: delete ‘$\beta$’ so it reads ‘($\sigma^2 = 0$)’

p.193 lines -3, -8; and p.194 line 7: “cocommutative” should be “coassociative”

p.196 line -4: If $H$ is in the center of $G$ and $A$ is a trivial $G$-module then $G/H$ acts trivially ...

p.197 Example 6.8.5: The two occurrences of $D_{2m}$ should read $D_m$ (on the first and last lines).

p.201 line 11: $f = f'$ should be $f_1 = f_2$

p.201 Exercise 6.9.2: If ... and ... are central extensions, and $X$ is perfect, show ...

p.203 lines 1–2: When $F_q$ is a finite field, and $(n, q) \neq (2, 2), (2, 3), (2, 4), (2, 9), (3, 2), (3, 4), (4, 2)$, we know that $H_2(SL_n(\mathbb{F}_q); \mathbb{Z}) = 0$ [Suz, 2.9]. With these exceptions, it follows that

p.206 line 8: $H_q(S_n(X) \otimes_{\mathbb{Z}} A)$ should be $H_q(G; S_n(X) \otimes_{\mathbb{Z}} A)$

p.213 Exercise 6.11.11: ... Show that for $i \neq 0$:

$$H^i(G; \mathbb{Z}) = \begin{cases} \mathbb{Z}_p^\infty & i = 2 \\ 0 & \text{else.} \end{cases}$$

p.226: Line 3 of Exercise 7.3.5 should read: $\delta$-functors (assuming that that $k$ is a field, or that $N$ is a projective $k$-module):

p.238 lines 4–7: Replace these two sentences (Show that...it suffices to show that ...= 0.) by:

Conversely, suppose that $g = f/\mathfrak{f}$ for some free Lie algebra $f$ with $\mathfrak{f} \subseteq [f, f]$, and $g$ is free as a $k$-module. Show that if $H^2(g, M) = 0$ for all $g$-modules $M$ then $g$ is a free Lie algebra. Hint: It suffices to show that ...= 0.

p.255 line -8: $0 \leq i_s \leq \cdots \leq i_1 \leq m$ should be $0 \leq i_s < \cdots < i_1 \leq m$

p.256 line 8: identity (not identify)

p.257 line 15 (display): $\alpha_s(t)$ should be $\alpha_s(s)$

p.258 lines 1, 20 and -7: ‘combinational’ should be ‘combinatorial’

p.261 lines -9,-8,-6: ‘combinatorial’ is misspelled three more times

p.262 line 18: $g_r = g(\sigma_r u)^{-1}$

p.262 line 13: ‘every $n$’ should be ‘every sufficiently large $n$’

p.262 line -6: $\partial_i(y)$ should be $\partial_i(y) = x_i$

p.264 lines 2–3: the two occurrences of $S(X)$ should read $S(|X|)$

p.265 add to end of Exercise 8.3.3:

Extend exercise 8.2.5 to show that a homomorphism of simplicial groups $G \to G''$ is a Kan fibration if and only if the induced maps $N_n G \to N_n G''$ are onto for all $n > 0$. In this case there is also a long exact sequence, ending in $\pi_0(G'')$.

p.266 line 14: that $\sigma_j(x_i) \neq 0$, then $y = y - \sigma_j \partial_i y = \sum_{j<i} \sigma_j(x'_j)$. By induction, $y = 0$. Hence $D_n \cap N_n = 0$.

p.267 line 5: fix subscript on sum: $d\sigma_p(x) = \sum_{p+2}^n$

p.267 line 6: $d\sigma^2_p(x) + \sigma_p d\sigma_p(x) = \sum_{i=p+2}^{n+1} + \sum_{i=p+2}^n$

p.267 line 7: $= (-1)^p \sigma_p(x)$

p.267 line 8: Hence $\{s_n = (-1)^p \sigma^p\}$

p.268 line -3: ‘$\{0, 1, \ldots, i - 1\}$’ should be ‘$\{0, 1, \ldots, i\}$’

p.273: the middle display should read

$$\text{Hom}_{Ch}(N A, C) \cong \text{Hom}_{S, A}(A, K(C)).$$
p.278 display on line 8: 1 should be subtracted from the subscripts: $\sigma^h_{\mu(n)-1} \cdots \sigma^h_{\mu(p+1)-1} \sigma^v_{\mu(p)-1} \cdots \sigma^v_{\mu(1)-1}$

p.280 lines 10–11: $\eta$: $1_c \to UF$ and ... $\varepsilon$: $FU \to 1_B$. (switch $B$ and $C$)

p.283 line -2: $\bar{r}_{i+1}$ should be $\bar{r}_i r_{i+1}$

p.287 lines -5, -4: “is an exact sequence” should be “is a sequence” and “is also exact” should be “is exact”

p.290 line before 8.7.9: Insert sentence: The proof of Theorem 3.4.3 goes through to prove that $\text{Ext}^1_{R/k}(M, N)$ classifies equivalence classes of $k$-split extensions of $M$ by $N$.

p.291 line -2: ... to $(R/I)^d$. If each $x_i R \subset R$ is $k$-split then:

p.294 line -8: If $M$ is an $R$-module, (‘a $k$’ should be ‘an $R$’)

p.295 line -4 (display): $D^1(R, M)$ should be $D^1(R/k, M)$

p.296 8.8.6: “If $k$ is a field” should be “If $R$ is a field”

p.297, line -9: the sequence should read

$$\cdots \to D_{n+1}(R/K, M) \to D_n(K/k, M) \to D_n(R/k, M) \to D_n(R/K, M) \to D_{n-1}(K/k, M) \to \cdots.$$

p.298 line 2 of 8.8.7: Commalg should be in roman font: Commalg

p.301 lines 4–5: the ranges should be “if $0 < i < n$” and “if $i = n + 1$” respectively.

p.304 Exercise 9.1.3: the variable $n$ should $m$ each time $(y_n, R^n, n, p < n)$; and $x$ (on line 17) should be $x$

p.307 line -1 should read:

As $\text{Tor}_{1}^{R'/k}(R^e, M) = 0$, the long exact relative Tor sequence (Lemma 8.7.8) yields

p.322: On line 1, insert “If $1/2 \in k$, ” before “$\Omega^*_{R/k}$ is the free graded-commutative” and (on line 4) add the sentence: In general, $\Omega^*_{R/k}$ is the free alternating $R$-algebra generated by $\Omega^1_{R/k}$.

p.325 line -6: $(-1)^n$ should be $(-1)^n p$


p.355 line 7: $u = \alpha_n$ should read ”... $u = \alpha_n+1$”

p.355 line -7: the final term in the display should be $HC_{n-2}(R)$

p.359 Exercise 9.9.5: This is wrong; replace it with:

**Exercise 9.9.5** (Grauert-Kerner) Consider the artinian algebra $R = k[x, y]/(\partial f/\partial x, \partial f/\partial y, x^5)$, where $f = x^4 + x^2y^3 + y^5$. Show that $I = (x, y)R$ is nilpotent, and $f$ is a nonzero element of $H^0_{dR}(R)$ which vanishes in $H^0_{dR}(R/R/I)$.

p.370 line -6: [ho-]“mopy” should be [ho-]“motopy” and $b''$ should be $b''$

p.375 line 4: $u\gamma = ig : B' \to B$ should be $T(u)\gamma = ig : B' \to T(B)$

p.376 line 8: after “naturality of the mapping cone construction” add “and the chain homotopy between $gu$ and $uf$.

p.376 line -10: diagram ... commutes up to chain homotopy.

p.381 line 6: ‘(left)’ should be ‘(right)’

p.382 lines 6-7 (10.3.8): the two occurrences of ‘right’ should be ‘left’

p.383 line 7 (Corollary 10.3.10): after ‘object’ add ’, and that $S$ is saturated.’ Add to the end of the proof the sentence: “So $S$ contains maps $Y \xrightarrow{0} X \xrightarrow{0} Z$, and hence $X \xrightarrow{0} X$.”

p.384 lines 9, 11: ‘left’ fraction should be ‘right’ fraction

p.384 line 13: Replacing ‘right’ by ‘left’

p.385 line 1: $B$ is a small category and $\text{Ext}(A, B)$ is a set for all $A \in A$, $B \in \mathcal{B}$. Then show that...

p.386 lines 7–9: six occurrences of $g$ should be $v$: ...there should be a $v : X \to Z$... $f - g = uv$. Embed $v$ in an exact triangle $(t, v, w)$... Since $vt = 0$, $(f - g)t = uvw = 0$, ...
p.386 lines 14–18: Replace the two sentences “Given $us^{-1}_{1}: \ldots$ triangle in $K$” with: “The exact triangles in $S^{-1}K$ are defined to be those triangles which are isomorphic, in the sense of (TR1), to the image under $K \to S^{-1}K$ of an exact triangle in $K$.”

p.386 line 20: replace “straightforward but lengthy; one uses the fact” with “straightforward; one uses (TR3) and the fact that…”

p.387 line -12 (10.4.5): delete ‘well-powered’ (Gabber points out that this condition is superfluous).

p.388 lines 2–3: the ‘$-dky$’ and ‘$+dk$’ should be ‘$+dky$’ and ‘$-dk$’

p.396 line 10: [Hart, II.5]

p.396 line 12: [Hart, exercise III.6.4] should be [HartRD, II.1.2]

p.400 line 8: the last two $(A, B)$ should be $(A, T^n B)$

p.400 line -7: Hom$(-, B)$ should be Hom$(-, B)$

p.405 line -2: a natural homomorphism in $D(R)$, which is an isomorphism if either each $C_i$ is fin. gen. projective or else $A$ is quasi-isomorphic to a bounded below chain complex of fin. gen. projective $R$-modules:

p.420 line 13: in exercise 6.11.3 (not 6.11.4)

p.427 line -1: $I \in I$ should be $i \in I$

p.428 line 16: $F_i \to F_i \to C$ should be $F_j \to F_i \to C$

p.429 line 15: ’Chapter 1’ should read ’Chapter 2’

p.431 line 6: ’functions’ should be ’functors.’

p.435 under ‘AB4 axiom’: add page 55

p.439 add entry to Index under ‘double chain complex’:

Connes’ — $B$. See Connes’ double complex.

p.444, under ‘Lie group’: [page] 158 should be 159

p.445, line -6: ‘Ore’ should be ‘Ore’

p.448 column 2: lines 29-30 should only be singly indented (“— of” refers to spectral sequence)

References