3.6: The final tableau of the given LPP is

<table>
<thead>
<tr>
<th>1 2 1 1 0 0 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1 x2 x3 x4 x5 x6 x7</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>1 x3 4/9 0 1 -4/9 4/3 -4/9 0 4/3</td>
</tr>
<tr>
<td>2 x2 2/3 1 0 4/3 0 1/3 0 4</td>
</tr>
<tr>
<td>0 x7 13 0 0 -10 9 -2/3 -1/9 1 34/3</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>7/9 0 0 12/9 4/3 2/9 0 28/3</td>
</tr>
</tbody>
</table>

(a) If $c_1$ is changed from 1 to 3, the resulting tableau is

<table>
<thead>
<tr>
<th>3 2 1 1 0 0 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1 x2 x3 x4 x5 x6 x7</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>1 x3 4/9 0 1 -4/9 4/3 -4/9 0 4/3</td>
</tr>
<tr>
<td>2 x2 2/3 1 0 4/3 0 1/3 0 4</td>
</tr>
<tr>
<td>0 x7 13 0 0 -10 9 -2/3 -1/9 1 34/3</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>-11 9 0 0 14 9 1 3 2/3 0 28/3</td>
</tr>
</tbody>
</table>

which represents a nonoptimal solution. Note that changing the objective function coefficient of a nonbasic variable affects only the objective row entry under that variable; the only thing which can "go wrong" as a result of this change is for that one objective row entry to become negative. Changing the objective function coefficient of a basic variable could change other objective row entries; again the issue is whether any of these become negative.

In our example, one step of the simplex method produces an optimal solution in the next tableau:

<table>
<thead>
<tr>
<th>3 2 1 1 0 0 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1 x2 x3 x4 x5 x6 x7</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>3 x1 1 0 3/4 -1/4 3/4 -1/4 0 3</td>
</tr>
<tr>
<td>2 x2 0 1 -3 2 3/2 -1 1/2 0 2</td>
</tr>
<tr>
<td>0 x7 0 0 -13 4 -3 4 -7/4 1/4 1 7</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>0 0 11 4 5 4 1 4 0 13</td>
</tr>
</tbody>
</table>

The new optimal solution is $x_1 = 3, x_2 = 2, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 0, x_7 = 7$ with $z = 13$.

(b) Changing $b_2$ to 26 produces a new tableau, whose rightmost column, which represents the new values of the basic variables, is found by multiplying the current $B^{-1}$ by the revised resource vector $b$: 
Since this is not feasible solution, we use the dual simplex method to restore feasibility:

\[
\begin{bmatrix}
\frac{1}{3} & -\frac{1}{9} & 0 \\
0 & \frac{1}{3} & 0 \\
-\frac{2}{3} & -\frac{1}{9} & 1
\end{bmatrix}
\begin{bmatrix}
8 \\
26 \\
18
\end{bmatrix}
= 
\begin{bmatrix}
\frac{2}{3} \\
\frac{26}{3} \\
\frac{88}{9}
\end{bmatrix}
\]

becomes, after applying the dual simplex method (pivoting on the \(-\frac{1}{9}\) in row 1, column 6):

\[
\begin{bmatrix}
1 & 2 & 1 & 1 & 0 & 0 & 0 \\
1 & x_3 & \frac{4}{9} & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & -\frac{1}{9} & 0 & -\frac{2}{9} \\
2 & x_2 & \frac{2}{3} & 1 & 0 & \frac{4}{3} & 0 & \frac{1}{3} & 0 & \frac{26}{3} \\
0 & x_7 & \frac{13}{9} & 0 & 0 & -\frac{10}{9} & -\frac{2}{3} & -\frac{1}{9} & 1 & \frac{88}{9}
\end{bmatrix}
\begin{bmatrix}
4 \\
3 \\
4 \\
34
\end{bmatrix}
= 
\begin{bmatrix}
1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16
\end{bmatrix}
\]

The new optimal solution is \(x_1 = 0, x_2 = 8, x_3 = 0, x_4 = 0, x_5 = 0, x_6 = 2, x_7 = 10\) with \(z = 16\).

(c) Changing \(c_3\) from 1 to \(\frac{1}{2}\) has no effect on the optimal solution. The same would be true of any change in value of \(c_3\) between -1 and 5, inclusive, hence any value of \(c_3\) itself between 0 and 6.
Since \(x_3\) is a basic variable in the optimal solution, changing \(c_3\) does affect the objective function value, which is now

\[
\begin{bmatrix}
c_3 \\
c_2 \\
c_7
\end{bmatrix}
= 
\begin{bmatrix}
\frac{4}{3} \\
4 \\
\frac{34}{3}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{2} \\
2 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
\frac{26}{3}
\end{bmatrix}
\]

The optimal solution is still \(x_1 = 0, x_2 = 4, x_3 = \frac{4}{3}, x_4 = 0, x_5 = 0, x_6 = 0, x_7 = \frac{34}{3}\), but now with objective function value \(z = \frac{26}{3}\).

(d) If \(b_3\) is changed from 18 to 127, the new vector of values of basic variables is
\[
\begin{bmatrix}
\frac{1}{3} & -\frac{1}{9} & 0 \\
0 & \frac{1}{3} & 0 \\
-\frac{2}{3} & \frac{1}{9} & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ x_3
\end{bmatrix}
= 
\begin{bmatrix}
8 \\ 12 \\ 18
\end{bmatrix}
\begin{bmatrix}
x_4 \\ x_5 \\ x_6
\end{bmatrix} = 
\begin{bmatrix}
\frac{4}{3} \\ 4 \\ \frac{361}{3}
\end{bmatrix}
\]

This is still a feasible solution, so the basic variables are still \( x_3, x_2, \) and \( x_7 \), in that order. Moreover, the only change has been to the value of the slack variable \( x_7 \), which makes no contribution to the objective function. Our optimal solution is therefore

\[
x_1 = 0, x_2 = 4, x_3 = \frac{4}{3}, x_4 = 0, x_5 = 0, x_6 = 0, x_7 = \frac{361}{3} \text{ with } z = \frac{28}{3}.
\]

4.1: Let \( x_1 \) = the number of chairs with solid backs and seats manufactured per month  
\( x_2 \) = the number of chairs with ladder backs and seats manufactured per month  
\( x_3 \) = the number chairs with vinyl-covered backs and seats manufactured per month

Maximize the profit \( z = 10x_1 + 13x_2 + 8x_3 \) subject to

\[
\begin{align*}
x_1 + 1.2x_2 + .7x_3 & \leq 600 \text{ (sanding constraint)} \\
.5x_1 + .5x_2 + .3x_3 & \leq 300 \text{ (staining constraint)} \\
.7x_1 + .7x_2 + .3x_3 & \leq 300 \text{ (varnishing constraint)} \\
.7x_3 & \leq 140 \text{ (upholstering constraint)}
\end{align*}
\]

\( x_1, x_2, x_3 \geq 0 \) and integral