Formulas for Exam #2

$$\nabla f(x,y,z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$

$$D = f_{xx}f_{yy} - f_{xy}^2 \qquad \begin{array}{llll} D>0 & \text{and} & f_{xx}>0 & \text{implies local minimum} \\ D>0 & \text{and} & f_{xx}<0 & \text{implies local maximum} \\ D<0 & & & \text{implies saddle point} \end{array}$$

Center of mass - 2 dimensions

$$\text{mass} = m = \iint_R \rho(x,y)\,dV \qquad (\bar{x},\bar{y}) = \frac{1}{m}(M_y, M_x)$$

$$M_y = \iint_R x\,\rho(x,y)\,dA \quad \text{and} \quad M_x = \iint_R y\,\rho(x,y)\,dA$$

Center of mass - 3 dimensions

$$\text{mass} = m = \iiint_E \rho(x,y,z)\,dV \qquad (\bar{x},\bar{y},\bar{z}) = \frac{1}{m}(M_{yz}, M_{xz}, M_{xy})$$

$$M_{yz} = \iiint_E x\,\rho(x,y,z)\,dV, \quad M_{xz} = \iiint_E y\,\rho(x,y,z)\,dV, \quad M_{xy} = \iiint_E z\,\rho(x,y,z)\,dV$$

Center of mass - a wire in the plane

$$\text{mass} = m = \int_C \rho(x,y)\,ds \qquad (\bar{x},\bar{y}) = \frac{1}{m}(M_y, M_x)$$

$$M_y = \int_C x\,\rho(x,y)\,ds \quad \text{and} \quad M_x = \int_C y\,\rho(x,y)\,ds$$

Polar coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad dA = r\,drd\theta$$

Cylindrical coordinates:

$$x = r\cos(\theta), \qquad y = r\sin(\theta), \qquad z = z, \qquad dV = r\,drd\theta dz$$

Spherical coordinates:

$$x = \rho\sin(\varphi)\cos(\theta), \qquad y = \rho\sin(\varphi)\sin(\theta), \qquad z = \rho\cos(\varphi), \qquad dV = \rho^2\sin(\varphi)\,d\rho d\theta d\varphi$$

Change of variables:

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \iiint_S f(x(u,v,w),y(u,v,w),z(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|\,du\,dv\,dw$$

$$\text{Line Integrals}: \qquad ds = |\mathbf{r}'(t)|dt \qquad dx = x'(t)dt \qquad d\mathbf{r} = \mathbf{r}'(t)dt$$