Formulas for the Final Exam

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$
$$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$$
$$\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{(\mathbf{a}\cdot\mathbf{b})}{(\mathbf{a}\cdot\mathbf{a})}\mathbf{a}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$
$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$$

A line with direction $\mathbf{v}$ passing through the point $\mathbf{r_0}$: $\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$

The equation of the plane tangent to $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$:

$$z - f(x_0, y_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

The distance from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$:

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$s(t) = \int_a^t |\mathbf{r}'(u)|\,du \quad \text{and} \quad \frac{ds}{dt} = |r'(t)|$$

If $F(x, y, z) = K$, then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z}$

Curvature for $\mathbf{r}(t)$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

Curvature for a plane curve $y = f(x)$

$$\kappa = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}}$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \qquad \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$$

$D = f_{xx}f_{yy} - f_{xy}^2$

- $D > 0$ and $f_{xx} > 0$ implies local minimum
- $D > 0$ and $f_{xx} < 0$ implies local maximum
- $D < 0$ implies saddle point

CYLINDRICAL: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$, $dV = r\,dr d\theta dz$

SPHERICAL: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$, $dV = \rho^2\sin(\phi)\,d\rho d\theta d\phi$

CHANGE OF VARIABLES: $$\iint_R f(x, y)\,dx\,dy = \iint_S f(x(u, v), y(u, v))\left|\frac{\partial(x, y)}{\partial(u, v)}\right| du\,dv$$

$$ds = |r'(t)|dt, \quad dx = x'(t)dt, \quad d\mathbf{r} = \mathbf{r}'(t)dt, \quad dS = |\mathbf{r}_u\times\mathbf{r}_v|\,dA = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA,$$

$$d\mathbf{S} = \mathbf{n}\,dS = \pm(\mathbf{r}_u\times\mathbf{r}_v)\,dA, \quad \mathbf{n} = \pm\frac{\mathbf{r}_u\times\mathbf{r}_v}{|\mathbf{r}_u\times\mathbf{r}_v|} = \pm\frac{\nabla F}{|\nabla F|} = \pm\frac{1}{\sqrt{1 + (f_x)^2 + (f_y)^2}}\langle f_x, f_y, -1\rangle$$

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_D \mathbf{F}\cdot(\mathbf{r}_u\times\mathbf{r}_v)\,dA = \iint_D\left(-P\frac{\partial f}{\partial x} - Q\frac{\partial f}{\partial y} + R\right)dA$$

GREEN'S THEOREM: $$\int_C P\,dx + Q\,dy = \iint_D\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA$$

STOKES' THEOREM: $$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \text{curl}\,\mathbf{F}\cdot d\mathbf{S}$$

THE DIVERGENCE THEOREM: $$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \text{div}\mathbf{F}\,dV$$