Name

QUIZ 2 — Math 250:C2

1. (3 points) Answer True or False to each of the following questions.

a) If the reduced row echelon form of the augmented matrix of a system of linear equations contains a zero row, then the system is consistent. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ F

b) If the reduced row echelon form of $[A\ \mathbf{b}]$ constains a zero row, then $A\mathbf{x} = \mathbf{b}$ must have infinitely many solutions. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ F

c) There exists a 5×8 matrix with rank 3 and nullity 2. null(A) + rank(A) = # of columns of A F

2. (8 ponits) The reduced row echelon form of the augmented matrix of a system of linear equations is

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 6 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

a). Circle the pivot entries.

b). Determine whether the system is consistent. Yes.

c). If the answer to part b) is yes, determine the *basic* and *free variables* and express the general solution of the given system in vector form.

x_1, x_3 are basic variables; x_2, x_4 are free variables

$$\begin{cases} x_3 = 7 - 4x_4 \\ x_1 = 6 - 3x_2 + 2x_4 \end{cases}$$

In vector form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 - 3x_2 + 2x_4 \\ x_2 \\ 7 - 4x_4 \\ x_4 \end{bmatrix}$

$$= \begin{bmatrix} 6 \\ 0 \\ 7 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -4 \\ 1 \end{bmatrix}.$$

turn over

3. (9 points) Find a row echelon form for the following system

$$x_1 + rx_2 = 5$$
$$3x_1 + 6x_2 = s$$

and determine the values of r and s for which the system has (a) no solutins, (b) exactly one solution, and (c) infinitely many solutions.

$$\begin{bmatrix} 1 & r & 5 \\ 3 & 6 & s \end{bmatrix} \xrightarrow{(-3)r_1+r_2} \begin{bmatrix} 1 & r & 5 \\ 0 & 6-3r & s-15 \end{bmatrix}$$

(a) System has no sol'n when $6-3r=0$, and $s-15 \neq 0$, i.e. when $r=2$, and $s \neq 15$.

(b) System has exactly one sol'n when $6-3r \neq 0$, i.e. when $r \neq 2$.

(c) System has ∞ many sol'ns when $6-3r = s-15 = 0$, i.e. when $r=2$ and $s=15$.

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QUIZ 3 — Math 250:C2

1. (4 points) Answer True or False to each of the following questions.

   a) If $\mathcal{S}_1$ and $\mathcal{S}_2$ are finite subsets of $\mathcal{R}^n$ having equal spans, then $\mathcal{S}_1 = \mathcal{S}_2$. F

   b) Let $\mathcal{S}$ be a nonempty set of vectors in $\mathcal{R}^n$, and let $\mathbf{v}$ be in $\mathcal{R}^n$. Then span of $\mathcal{S}$ and $\mathcal{S} \cup \{\mathbf{v}\}$ are equal if and only if $\mathbf{v}$ is in $\mathcal{S}$. F

   c) If the only solution to $A\mathbf{x} = \mathbf{0}$ is $\mathbf{0}$, then the nullity of A is 0. T

   d) The columns of a 3×4 matrix are linearly dependent. T

2. (4 points) Determine whether the set

$$\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \right\}$$

is a generating set for $\mathbb{R}^3$.

$$\begin{bmatrix} -1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since the last row of the echelon form is $(0\ 0\ 0)$, the given set is NOT a generating set for $\mathbb{R}^3$

turn over to continue

3. (12 ponits) Given the set $\mathcal{S}$ of vectors

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

a). Find a subset of $\mathcal{S}$ with the same span as $\mathcal{S}$ that is as small as possible.

$$\begin{bmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 0 \end{bmatrix} \to \begin{bmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 1 \end{bmatrix} \to \begin{bmatrix} -1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

So we can keep the pivot columns:

$\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ produces the same span as S.

b). Write the vector(s) eliminated in part a) as a linear combination of the remaining vectors.

From (a), to solve $x_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$,

it suffices $\begin{cases} -x_1 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + x_4 = 0 \\ x_4 = 0 \end{cases}$ (x_3 = free).

Set $x_3 = 1$, we find $\begin{cases} x_1 = 1 \\ x_2 = -2 \\ x_3 = 1 \\ x_4 = 0 \end{cases}$, i.e. $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + (-2) \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + 1 \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = (-1) \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

c). Verify that the remaining vectors from part a) is linearly independent.

If $x_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, then

using the echelon form in (a) $\Rightarrow \begin{cases} -x_1 + x_4 = 0 \\ x_2 + x_4 = 0 \\ x_4 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_4 = 0 \end{cases}$

So the remaining set is l.i.