Name

QUIZ 4 — Math 250:C2

1. (4 points) Answer True or False to each of the following questions.

   a) If A and B are invertible $n \times n$ matrices, then AB^T is invertible. ✓

   b) If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible. X

   c) If the $n \times n$ matrices A, B, and C satisfy $AC = BC$, and $C \neq 0$, then $A = B$. X

   $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = B$, $C = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}$, $AC = BC$, yet $A \neq B$.

   d) If every column of an $m \times n$ matrix A contains a pivot position, then the matrix equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b} \in \mathbb{R}^m$. X

   $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

2. (4 ponits) Determine, if possible, a value of r for which the set of vectors

$$\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ r \end{bmatrix} \right\}$$

is linearly dependent.

$$\begin{bmatrix} -2 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & -3 & r \end{bmatrix} \xrightarrow{\frac{1}{2}r_1 + r_3} \begin{bmatrix} -2 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & -\frac{5}{2} & r - \frac{1}{2} \end{bmatrix}$$

$$\xrightarrow{\frac{5}{2}r_2 + r_3} \begin{bmatrix} -2 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & r - \frac{1}{2} + \frac{5}{2} \end{bmatrix}$$

The given vectors are l.d when there are fewer than 3 pivots. In this case, the condition is $r - \frac{1}{2} + \frac{5}{2} = 0$, i.e. $r = -2$.

turn over to continue.

3. (4 points) Compute the entries in the first row of the resulting matrix from the product

$$\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}^T \begin{bmatrix} 3 & 8 & 1 \\ 2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 8 & 1 \\ 2 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot 3 + 3\cdot 2 & 1\cdot 8 + 3\cdot 0 & 1\cdot 1 + 3\cdot 4 \\ \times & \times & \times \end{bmatrix} = \begin{bmatrix} 9 & 8 & 13 \\ \times & \times & \times \end{bmatrix}$$

4. (8 points) Given the elementary matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

a) Describe the elementary row operation that gives rise to E, then compute

$$E \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d+2g & e+2h & f+2i \\ g & h & i \end{bmatrix}.$$

$2r_3 + r_2 \to r_2$ is the row operation that gives rise to E.

Performing the same op. on $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, we get

b) Find E^{-1}.

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$