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Name

QUIZ 9 — Math 250:09

1. (4 points) Answer True or False to each of the following questions.

   a) If a vector $\mathbf{u}$ is orthogonal to vectors $\mathbf{v}$ and $\mathbf{w}$, then it is orthogonal to all vectors in the *span*$\{\mathbf{v}, \mathbf{w}\}$. T

   b) The property $||\mathbf{v} + \mathbf{w}||^2 = ||\mathbf{v}||^2 + ||\mathbf{w}||^2$ holds *only* for a pair of orthogonal vectors $\mathbf{v}$ and $\mathbf{w}$. T

   c) If $\mathbf{Proj}_L\mathbf{u}$ represents the orthogonal projection of $\mathbf{u}$ onto the straightline L, then $\mathbf{Proj}_L\mathbf{u} \perp \mathbf{v}$ for all $\mathbf{v}$ in L. F

   $\vec{u} - \mathrm{Proj}_L(\vec{u})$, not $\mathrm{Proj}_L(\vec{u})$, $\perp \vec{v}$.

   d) Every orthonormal subset is linearly independent. T

2. (6 points) Suppose that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$ such that $||\mathbf{u}|| = 3$, $||\mathbf{v}|| = 2$, $||\mathbf{w}|| = 5$, $\mathbf{u} \cdot \mathbf{v} = 1$, $\mathbf{u} \cdot \mathbf{w} = -2$, and $\mathbf{v} \cdot \mathbf{w} = -3$. Compute $||2\mathbf{u} - 3\mathbf{v}||^2$ and $(2\mathbf{u} - 3\mathbf{v}) \cdot \mathbf{w}$.

$$||2u-3v||^2 = (2u-3v)\cdot(2u-3v)$$
$$= 4u\cdot u - 6u\cdot v - 6v\cdot u + 9v\cdot v$$
$$= 4\cdot 9 - 6\cdot 1 - 6\cdot 1 + 9\cdot 4$$
$$= 60$$

$$(2u-3v)\cdot w = 2u\cdot w - 3v\cdot w$$
$$= 2(-2) - 3(-3) = 5$$

turn over to continue.

3. (10 points) Given the vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ -3 \\ -1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 0 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

a) Compute the dot products $\mathbf{u}\cdot\mathbf{v}$, $\mathbf{u}\cdot\mathbf{w}$, $\mathbf{v}\cdot\mathbf{w}$ and the norm of each of the three vectors.

$u \cdot v = 2 + 6 + (-6) + 0 = 2, \quad u \cdot w = 1 + 2 + (-3) + (-1) = -1,$

$v \cdot w = 2 + 3 + 2 + 0 = 7.$

$u \cdot u = 1 + 4 + 9 + 1 = 15, \quad v \cdot v = 2^2 + 3^2 + 2^2 = 17, \quad w \cdot w = 4$

So $\|\vec{u}\| = \sqrt{15}, \quad \|v\| = \sqrt{17}, \|w\| = \sqrt{4} = 2.$

b) Construct an orthogonal basis for $span\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

$k_1 = u, \quad k_2 = v - \frac{v \cdot k_1}{k_1 \cdot k_1} k_1 = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 0 \end{bmatrix} - \frac{2}{15}\begin{bmatrix} 1 \\ 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{28}{15} \\ \frac{41}{15} \\ \frac{36}{15} \\ +\frac{2}{15} \end{bmatrix}$

$k_3 = w - \frac{w \cdot k_1}{k_1 \cdot k_1} k_1 - \frac{w \cdot k_2}{k_2 \cdot k_2} k_2$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{-1}{15}\begin{bmatrix} 1 \\ 2 \\ -3 \\ -1 \end{bmatrix} - \frac{\frac{28+41+36+2}{15}}{\frac{28^2+41^2+36^2+2^2}{15^2}} \begin{bmatrix} \frac{28}{15} \\ \frac{41}{15} \\ \frac{36}{15} \\ \frac{2}{15} \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{15}\begin{bmatrix} 1 \\ 2 \\ -3 \\ -1 \end{bmatrix} - \frac{28+41+36+2}{28^2+41^2+36^2+2^2}\begin{bmatrix} 28 \\ 41 \\ 36 \\ 2 \end{bmatrix}$