

Problem MAA11274 (Knuth)

Calculate $S(m,n) = \sum(2^k \cdot \text{binomial}(2m-k, m+n), k=0..m) + \sum(\text{binomial}(2m+1, m+j), j=1..n)$

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> assume(n::natural, m::natural):  
  term1 := (m, n) -> 2^k * binomial(2*m-k, m+n):  
  term2 := (m, j) -> binomial(2*m+1, m+j):  
  S := (m, n) -> sum(term1(m, n), k=0..m) + sum(term2(m, j), j=1..n):
```

Step 1: Show that $S(m,n)$ is independent of n , that is, $S(m, n+1) - S(m, n) = 0$.

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> simplify(sum(term1(m, n+1) - term1(m, n), k=0..m) + term2(m, n+1), GAMMA);  
0
```

Step 2: Hence, $S(m,n) = S(m,m)$, that is

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> simplify(S(m, m));  
4^m
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