640-Experimental Math
11/20/16

Homework 21 (Part 5)

5) We want to show that the hook length formula is equivalent to

$$F(a_1, ..., a_k) = \frac{\left(\prod_{i=1}^{k-1}\prod_{j=i+1}^{k}(a_i - a_j + j - i)\right) \cdot \left(\sum_{i=1}^{k} a_i\right)!}{\left(\prod_{i=1}^{k-1}\prod_{j=1}^{k-i}(a_i + j)\right) \cdot \prod_{i=1}^{k} a_i!}.$$

Since $\sum_{i=1}^{k} a_i = n$, we have to show that

$$\prod h_\lambda(i,j) = \frac{\left(\prod_{i=1}^{k-1}\prod_{j=1}^{k-i}(a_i + j)\right) \cdot \prod_{i=1}^{k} a_i!}{\left(\prod_{i=1}^{k-1}\prod_{j=i+1}^{k}(a_i - a_j + j - i)\right)} = \prod_{i=1}^{k-1} \frac{(a_i + k - i)!}{\prod_{j=i+1}^{k}(a_i - a_j + j - i)}$$

where $h_\lambda(i,j)$ is the hook length of box j in row i, and the product is taken over all boxes in the partition. For a fixed i, the numerator is the product of all numbers between the maximum possible hook length for an element of the row i, a_i+k-i, and 1. The denominator determines which of these lengths do not actually appear in row i.