1 Problem 1

All but one row of the Jacobian matrix of an elementary transformation in n-dimensional space equals the corresponding row of I_n. Call the exception row k: then column k has entry a and some column $m \neq k$ has entry $r \cdot bx_m^{r-1}$. By the laws of determinants, the Jacobian determinant is

$$1 \cdot a + 0 = a$$

corresponding to the composition of row multiplication in row multiple addition applied to I_m.

2 Problem 2

By the Inverse Function Theorem, since an elementary transformation is continuously differentiable it is invertible if its Jacobian determinant is nonzero – *i.e.,* if $a \neq 0$. Moreover, its inverse has Jacobian the inverse of the Jacobian matrix.

From the matrix that yields the opposite transformations to those of Problem 1, we obtain

$$\frac{\partial f_k}{\partial x_k} = \frac{1}{a} \tag{1}$$

$$\frac{\partial f_k}{\partial x_m} = -r \cdot bv^{r-1} \tag{2}$$

where the indices retain their meanings from Problem 1. Hence the inverse transformation is

$$(x_1, x_2, ..., \frac{1}{a}x_k - b \cdot x_m^{r-1}, ..., x_n)$$