

Dr. Z's Math151 Handout #3.2 [The Product and Quotient Rule]

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Problem Type 3.2.1 : Differentiate $f(x) = \text{Expression}_1(x) \times \text{Expression}_2(x)$, where both expressions are 'easy' to differentiate from known rules.

Example Problem 3.2.1: Differentiate $f(x) = x^4e^x$.

Steps

Example

1. Use the product rule $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

1. $(x^4e^x)' = (x^4)'e^x + (x^4)(e^x)' = 4x^3e^x + x^4e^x$

2. Use algebra to simplify.

2. Ans.: $(4x^3 + x^4)e^x$.

Problem Type 3.2.2 :

Differentiate

$$f(x) = \frac{\text{Expression}_1(x)}{\text{Expression}_2(x)} .$$

where both expressions are ‘easy’ to differentiate from known rules.

Example Problem 3.2.2: Differentiate

$$y = \frac{t^3 + t}{t^4 - 2} .$$

Steps

1. Use the quotient rule

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} .$$

2. Use algebra to simplify.

Example

- 1.

$$\left(\frac{t^3 + t}{t^4 - 2}\right)' = \frac{(t^3 + t)'(t^4 - 2) - (t^3 + t)(t^4 - 2)'}{(t^4 - 2)^2} =$$

$$\frac{(3t^2 + 1)(t^4 - 2) - (t^3 + t)(4t^3)}{(t^4 - 2)^2} =$$

$$\frac{(3t^6 - 6t^2 + t^4 - 2) - (4t^6 + 4t^4)}{(t^4 - 2)^2}$$

- 2.

$$-\frac{t^6 + 6t^2 + 3t^4 + 2}{(t^4 - 2)^2}$$