

Dr. Z's Math151 Handout #4.1 [Maximum and Minimum Values]

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**Problem Type 4.1.1** : Find the critical numbers of the function  $f(x) = \text{Expression}(x)$ .

**Example Problem 4.1.1**: Find the critical numbers of the function  $f(x) = x^3 + 3x^2 - 24x$ .

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**Steps**

**Example**

1. Find the derivative of  $f(x)$ , and set it equal to 0.

1.

$$f'(x) = (x^3 + 3x^2 - 24x)' =$$

$$3x^2 + 6x - 24 \quad .$$

We have to solve  $3x^2 + 6x - 24 = 0$ .

2. Solve this equation, by any method.

2.  $3x^2 + 6x - 24 = 0$  is the same as

$$3(x^2 + 2x - 8) = 0 \quad ,$$

which is the same as

$$3(x + 4)(x - 2) = 0 \quad .$$

There are two roots  $x = -4$  and  $x = 2$ .

3. If applicable, find when  $f'(x)$  is undefined (usually because it blows up, for example for  $f(x) = x^{1/2}$ ,  $f'(x) = x^{-1/2} = 1/x^{1/2}$ , and  $f'(x)$  is undefined at  $x = 0$ , so in that case 0 would be a critical point.)

3.  $f'(x) = 3x^2 + 6x - 24$  is always defined, so

**Ans.:** The set of critical numbers is  $\{-4, 2\}$ .

In our problem  $f'(x)$  is always defined (since it is a polynomial), so this other kind of critical numbers do not show up. The critical numbers are obtained by combining these two kinds.

**Problem Type 4.1.2 :** Find the absolute maximum and absolute minimum values of  $f(x) = \text{Expression}(x)$  on the closed interval  $[a, b]$ .

**Example Problem 4.1.2:** Find the absolute maximum and absolute minimum values of  $f(x) = x^3 - 3x + 2$  on the interval  $[0, 2]$ .

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**Steps**

1. Find the critical numbers of  $f(x)$  by solving  $f'(x) = 0$ .

2. The slate of **finalists** consists of those critical numbers from step 1 that **belong** in the specified interval **plus** the **endpoints**  $a$  and  $b$ .

3. For each of the finalists, plug-in into  $f(x)$ , and compare the scores. The smallest of these is the **absolute minimum value** and The largest of these is the **absolute maximum value**.

**Note:** Sometimes they may ask about the **absolute minimum** and **absolute maximum** of the function in the given interval. These are the corresponding locations. Here the absolute minimum is at  $x = 1$  and the absolute maximum is at  $x = 2$ .

**Example**

1.

$$(x^3 - 3x + 2)' = 3x^2 - 3 =$$

$$3(x - 1)(x + 1) \quad .$$

Solving  $3(x - 1)(x + 1) = 0$  gives the critical numbers  $x = -1$  and  $x = 1$ .

2.  $x = -1$  does not belong to the interval  $[0, 2]$  but  $x = 1$  does. The set of **finalists** in the contest is  $\{0, 1, 2\}$  consisting of the two endpoints 0 and 2 and the critical number  $x = 1$  that lies in the interval  $[0, 2]$ .

3.  $f(x) = x^3 - 3x + 2$ . So

$$f(0) = 2 \quad ,$$

$$f(1) = 1^3 - 3 \cdot 1 + 2 = 0 \quad ,$$

$$f(2) = 2^3 - 3 \cdot 2 + 2 = 4 \quad .$$

It follows that the **absolute minimum value** is 0 and the **absolute maximum value** is 4 .