Dr. Z's Math151 Handout #4.10 [Antiderivatives]

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Problem Type 4.10.1 : Find the most general antiderivative of the function $f(x)$.

Example Problem 4.10.1: Find the most general antiderivative of the function $f(x) = 5e^x + 8\sec^2 x$.

Steps	**Example**
1. You have to memorize the differentiation table in reverse. An antiderivative of x^n if $x^{n+1}/(n+1)$ (except when $n = -1$). The antiderivative of $\cos x$ is $\sin x$, The antiderivative of $\sin x$ is $-\cos x$, The antiderivative of $\sec^2 x$ is $\tan x$, etc.	**1.** An antiderivative of e^x is e^x, and that of $\sec^2 x$ is $\tan x$. Hence an antiderivative of $f(x)$ is $5e^x + 8\tan x$.
2. To find the **most general antiderivative**, you add C (an **arbitrary constant**) to the above answer.	**2. Ans.**: $5e^x + 8\tan x + C$.

Problem Type 16.2 : Find f if $f'(x) = Expression(x)$ and $f(a) = Number$.

Example Problem 16.2: Find f if $f'(x) = 6x - 2/x^2, x > 0$ and $f(1) = 4$.

Steps	**Example**
1. Find the most general antiderivative, like in problem 4.10.1, featuring C.	**1.** $f'(x) = 6x - 2/x^2$ so $f(x) = 3x^2 + 2x^{-1} + C$
2. Plug-in $x = a$ and solve for C the equation $f(a) = Number$.	**2.** $f(x) = 3x^2 + 2x^{-1} + C$, so $f(1) = 3 \cdot 1^2 + 2/1 + C = 5 + C$. But, on the other hand, from the data, $f(1) = 4$. So we have the equation $5 + C = 4$. Solving it yields $C = -1$.
3. Plug-in the specific C that you got in step 2 into the answer of step 1.	**3. Ans.**: $f(x) = 3x^2 + 2x^{-1} - 1$.