Dr. Z's Math151 Handout #5.4 [Indefinite Integrals and the Net Change Theorem]

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Problem Type 5.4.1 : Find the general indefinite integral

$$\int f(Var)dVar \quad .$$

Example Problem 5.4.1: Find

$$\int (1-t)(2+t^2)dt \quad .$$

Steps

1. Use algebra and/or trig. to simplify the integrand $f(Var)$ as much as possible.

2. Use the addition rule of integration to split the integral into its constituent parts, and then find an antiderivative for each piece using the table in your head ($\int t^n dt = t^{n+1}/(n+1)$, etc.). Do not bother with the C in the intermediate steps.

3. Add $+C$ to the answer. C stands for *arbitrary constant.*

Example

1. $(1-t)(2+t^2) = 2-2t+t^2-t^3$, so

$$\int (1-t)(2+t^2)dt = \int (2-2t+t^2-t^3)dt \quad .$$

2.

$$\int (2-2t+t^2-t^3)dt = 2\int 1dt - 2\int tdt + \int t^2dt - \int t^3dt \quad .$$

$$= 2t - 2(t^2/2) + t^3/3 - t^4/4 \quad .$$

3.

$$\textbf{Ans.} = 2t - t^2 + \frac{t^3}{3} - \frac{t^4}{4} + C \quad .$$

Problem Type 5.4.2 : Evaluate the (definite) integral

$$\int_a^b f(Var) d\, Var$$

Example Problem 5.4.2: Evaluate the (definite) integral

$$\int_{-1}^{1} (u^5 - u^3 + u^2) d\, u$$

Steps

1. First find the *indefinite integral*, like we did in 5.4.1, and get an expression in Var, except that you need not bother with the C.

$$\int f(u) d\, u = F(u)$$

2. Now stick the *limit of integrations* on the left, and an *evaluation line* on the right with the corresponding limits.

$$\int_a^b f(u) d\, u = F(u)|_b^a$$

3. Compute $F(b) - F(a)$ by plugging-in the upper limit (b) and substracting from it F plugged-in the lower limit (a).

Example

1.

$$\int (u^5 - u^3 + u^2) d\, u = \frac{u^6}{6} - \frac{u^4}{4} + \frac{u^3}{3}$$

2.

$$\int_{-1}^{1} (u^5 - u^3 + u^2) d\, u = \frac{u^6}{6} - \frac{u^4}{4} + \frac{u^3}{3}|_{-1}^{1}$$

3.

$$[\frac{(1)^6}{6} - \frac{(1)^4}{4} + \frac{(1)^3}{3}] - [\frac{(-1)^6}{6} - \frac{(-1)^4}{4} + \frac{(-1)^3}{3}] = \frac{2}{3} \quad .$$