

Dr. Z's Math151 Handout # 2.3 [Basic Limit Laws]

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**Problem Type 2.3.1:** Evaluate the limit using the Limit Laws

$$\lim_{x \rightarrow a} f(x) \quad ,$$

where  $f(x)$  is a **specific** function of  $x$ .

**Example Problem 2.3.1 :** Evaluate the limit using the Limit Laws

$$\lim_{x \rightarrow -4} (x^3 - 4x) \quad .$$

**Steps**

1. Use the appropriate rules at each stage mimicking the algebra. The “bottom lines” are  $\lim_{x \rightarrow c} x = c$  and  $\lim_{x \rightarrow c} k = k$  ( $k$  constant)

**Example**

1.

$$\lim_{x \rightarrow -4} (x^3 - 4x) = \lim_{x \rightarrow -4} x^3 - \lim_{x \rightarrow -4} 4x =$$

$$\left( \lim_{x \rightarrow -4} x \right)^3 - 4 \lim_{x \rightarrow -4} x = (-4)^3 - 4(-4) = -48 \quad .$$

**Ans.:** -48.

**Problem Type 2.3.2:** Given **general** (abstract) functions  $f(x)$  and  $g(x)$  and their limits at a certain point, evaluate the limits of algebraic expressions of them at that very same point.

**Example Problem 2.3.2 :** Assuming that  $\lim_{x \rightarrow 3} f(x) = 5$  and  $\lim_{x \rightarrow 3} g(x) = -2$ , evaluate the limits

$$(a) \quad \lim_{x \rightarrow 3} 2f(x) - 3g(x)$$

$$(b) \quad \lim_{x \rightarrow 3} f(x)g(x)$$

$$(c) \quad \lim_{x \rightarrow 3} \frac{f(x) + 1}{2g(x) + 1}$$

### Steps

1. Use the Limit Rules on the abstract functions, and at the end use the given data.

### Example

1.

$$(a) \quad \lim_{x \rightarrow 3} 2f(x) - 3g(x) = \lim_{x \rightarrow 3} 2f(x) - \lim_{x \rightarrow 3} 3g(x)$$

$$= 2 \left( \lim_{x \rightarrow 3} f(x) \right) - 3 \left( \lim_{x \rightarrow 3} g(x) \right) = 2 \cdot (5) - 3 \cdot (-2) = 16 \quad .$$

**Ans. to (a)** = 16 .

$$(b) \quad \lim_{x \rightarrow 3} f(x)g(x) = \left( \lim_{x \rightarrow 3} f(x) \right) \left( \lim_{x \rightarrow 3} g(x) \right)$$

$$= (5) \cdot (-2) = -10 \quad .$$

**Ans. to (b)** = -10 .

$$(c) \quad \lim_{x \rightarrow 3} \frac{f(x) + 1}{2g(x) + 1} = \frac{\lim_{x \rightarrow 3} (f(x) + 1)}{\lim_{x \rightarrow 3} (2g(x) + 1)}$$

$$\frac{\lim_{x \rightarrow 3} f(x) + \lim_{x \rightarrow 3} 1}{2(\lim_{x \rightarrow 3} g(x)) + \lim_{x \rightarrow 3} 1} =$$

$$\frac{(5 + 1)}{2 \cdot (-2) + 1} = \frac{6}{-3} = -2 \quad .$$

**Ans. to (c)**: -2.