Dr. Z's Math151 Handout #3.3 [The Product and Quotient Rule]

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Problem Type 3.3.1 : Differentiate $f(x) = Expression_1(x) \times Expression_2(x)$, where both expressions are 'easy' to differentiate from known rules.

Example Problem 3.3.1: Differentiate $f(x) = x^4 e^x$.

Steps	**Example**
1. Use the product rule $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.	**1.** $(x^4e^x)' = (x^4)'e^x + (x^4)(e^x)' = 4x^3e^x + x^4e^x$
2. Use algebra to simplify.	**2. Ans.**: $(4x^3 + x^4)e^x$.

Problem Type 3.3.2 :

Differentiate

$$f(x) = \frac{Expression_1(x)}{Expression_2(x)} \quad .$$

where both expressions are 'easy' to differentiate from known rules.

Example Problem 3.3.2: Differentiate

$$y = \frac{t^3 + t}{t^4 - 2} \quad .$$

Steps

1. Use the quotient rule

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \quad .$$

2. Use algebra to simplify.

Important comment: Sometimes you are told to differentiate a function, and it says "do not simplify". In that case, you don't need the present step.

Example

1.

$$\left(\frac{t^3 + t}{t^4 - 2}\right)' = \frac{(t^3 + t)'(t^4 - 2) - (t^3 + t)(t^4 - 2)'}{(t^4 - 2)^2}$$

$$= \frac{(3t^2 + 1)(t^4 - 2) - (t^3 + t)(4t^3)}{(t^4 - 2)^2}$$

2.

$$= \frac{(3t^6 - 6t^2 + t^4 - 2) - (4t^6 + 4t^4)}{(t^4 - 2)^2}$$

$$= \frac{3t^6 - 6t^2 + t^4 - 2 - 4t^6 - 4t^4}{(t^4 - 2)^2}$$

$$= \frac{-t^6 - 3t^4 - 6t^2 - 2}{(t^4 - 2)^2}$$

$$= -\frac{t^6 + 3t^4 + 6t^2 + 2}{(t^4 - 2)^2}$$