Dr. Z's Math151 Handout #3.5 [Higher Derivatives]

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Problem Type 3.5.1 : Find the first, second, and third derivatives of the function $f(x) = Expression(x)$.

Example Problem 3.5.1: Find the first, second, and third derivatives of the function $f(x) = x^4 + 3e^x + \sqrt{x}$.

Steps

1. These are really three different differentiation problems. You first find the derivative (a.k.a. the first derivative), and then you take the derivative of the derivative. Finally, to get the third derivative, you find the derivative of the second derivative. (And so on, if asked for.)

2. Now take the second derivative, i.e. the derivative of the answer from the previous step.

3. Now take the third derivative, i.e. the derivative of the answer from the previous step.

Example

1.

$$\frac{d}{dx}(x^4+3e^x+\sqrt{x}) = \frac{d}{dx}(x^4+3e^x+x^{1/2}) =$$

$$4x^3 + 3e^x + (1/2)x^{-1/2} \quad .$$

2.

$$\frac{d^2}{dx^2}(x^4+3e^x+\sqrt{x}) = \frac{d}{dx}(4x^3+3e^x+(1/2)x^{-1/2})$$

$$12x^2+3e^x+(1/2)(-1/2)x^{-3/2} = 12x^2+3e^x-(1/4)x^{-3/2}$$

3.

$$\frac{d^3}{dx^3}(x^4+3e^x+\sqrt{x}) = \frac{d}{dx}(12x^2+3e^x-(1/4)x^{-3/2})$$

$$24x+3e^x-(1/4)(-3/2)x^{-5/2} = 24x+3e^x+(3/8)x^{-5/2} \quad .$$

Problem Type 3.5.2 : An equation of motion is given, where s is in unit of distance and t is in units of time. Find (a) The times at which the acceleration is 0. (b) the displacement and velocity at these times.

Example Problem 3.5.2: As above with meters, second, and $s = t^4 - 4t^3 + 2$.

Steps

1. Find the velocity $v = ds/st$ and the acceleration $a = dv/dt = d^2s/dt^2$.

2. Set the acceleration a equal to 0, and solve for t.

3. Plug-in the value(s) of t found in step 2 into s of the problem, and v that you found in step 1.

Example

1.

$$v = \frac{d}{dt}(t^4 - 4t^3 + 2) =$$

$$4t^3 - 12t^2 \quad .$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(4t^3 - 12t^2) =$$

$$12t^2 - 24t \quad .$$

2. $12t^2 - 24t = 0$ hence $12t(t-2) = 0$, whose solutions are $t = 0$ and $t = 2$.

3. At $t = 0$:

$$s(0) = 0^4 - 4 \cdot 0^3 + 2 = 2$$

$$v(0) = 4 \cdot 0^3 - 12 \cdot 0^2 = 0 \quad .$$

At $t = 2$:

$$s(2) = 2^4 - 4 \cdot 2^3 + 2 = 16 - 32 + 2 = -14 \quad ,$$

$$v(2) = 4 \cdot 2^3 - 12 \cdot 2^2 = 32 - 48 = -16 \quad .$$

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