Dr. Z's Math151 Handout #4.9 [Antiderivatives]

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Problem Type 4.9.1 : Find the most general antiderivative of the function $f(x)$.

Example Problem 4.9.1: Find the most general antiderivative of the function $f(x) = 5e^x + 8\sec^2 x$.

Steps	**Example**
1. You have to memorize the differentiation table in reverse. An antiderivative of x^n if $x^{n+1}/(n+1)$ (except when $n = -1$). The antiderivative of $\cos x$ is $\sin x$, The antiderivative of $\sin x$ is $-\cos x$, The antiderivative of $\sec^2 x$ is $\tan x$, etc.	**1.** An antiderivative of e^x is e^x, and that of $\sec^2 x$ is $\tan x$. Hence an antiderivative of $f(x)$ is $5e^x + 8\tan x$.
2. To find the **most general antiderivative**, you add C (an **arbitrary constant**) to the above answer.	**2. Ans.**: $5e^x + 8\tan x + C$.

Problem Type 4.9.2 : Find f if $f'(x) = \mathit{Expression}(x)$ and $f(a) = \mathit{Number}$.

Example Problem 4.9.2: Find f if $f'(x) = 6x - 2/x^2, x > 0$ and $f(1) = 4$.

Steps	**Example**
1. Find the most general antiderivative, like in problem 4.9.1, featuring C.	**1.** $f'(x) = 6x - 2/x^2$ so $f(x) = 3x^2 + 2x^{-1} + C$
2. Plug-in $x = a$ and solve for C the equation $f(a) = \mathit{Number}$.	**2.** $f(x) = 3x^2 + 2x^{-1} + C$, so $f(1) = 3 \cdot 1^2 + 2/1 + C = 5 + C$. But, on the other hand, from the data, $f(1) = 4$. So we have the equation $5 + C = 4$. Solving it yields $C = -1$.
3. Plug-in the specific C that you got in step 2 into the answer of step 1.	**3. Ans.**: $f(x) = 3x^2 + 2x^{-1} - 1$.

A problem from a Previous Final (Spring 2008, #8 (10 points)).

Find $y = y(x)$ if $\frac{d^2y}{dx^2} = 4x$, $\frac{dy}{dx}(0) = 1$ and $y(0) = 0$.

Solution:

We know that $y'' = 4x$. To get y', we take the **anti-derivative** of y''

$$y' = \int 4x\,dx = 4\frac{x^2}{2} = 2x^2 + C \quad .$$

Next we need to find the value of C. Plugging-in $x = 0$, we get, since the problem tells us that $y'(0) = 1$

$$1 = 2 \cdot 0^2 + C$$

which is

$$1 = C$$

and this gives $C = 1$.

Intermediate answer: $y'(x) = 2x^2 + 1$.

To find $y(x)$, our ultimate goal, we find the **anti-derivative** of $2x^2+1$.

$$y(x) = \int (2x^2+1)\,dx = 2\frac{x^3}{3} + x + C = \frac{2x^3}{3} + x + C \quad ,$$

(note that this C is a **different** C than the one above).

To find this new C we plug-in $x = 0$. The problem tells us that $y(0) = 0$, so

$$0 = \frac{2 \cdot 0^3}{3} + 0 + C$$

which means

$$0 = C \quad .$$

So $C = 0$. Going back, above we have

$$y(x) = \frac{2x^3}{3} + x + 0 = \frac{2x^3}{3} + x \quad .$$

Final Ans.: $y(x) = \frac{2x^3}{3} + x$.