Dr. Z's Math151 Handout #5.3 [The Fundamental Theorem of Calculus, Part I]

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Problem Type 5.3.1 : Evaluate the (definite) integral

$$\int_a^b f(Var)d\,Var$$

Example Problem 5.3.1: Evaluate the (definite) integral

$$\int_{-1}^1 (u^5 - u^3 + u^2)d\,u$$

Steps	**Example**
1. First find the *antiderivative*, like we did in section 4.9, and get an expression in Var.	**1.** $$\int (u^5 - u^3 + u^2)d\,u = \frac{u^6}{6} - \frac{u^4}{4} + \frac{u^3}{3}$$
2. Now stick the *limit of integrations* on the left, and an *evaluation line* on the right with the corresponding limits. $$\int_a^b f(u)d\,u = F(u)\Big\vert_b^a$$	**2.** $$\int_{-1}^1 (u^5 - u^3 + u^2)d\,u = \frac{u^6}{6} - \frac{u^4}{4} + \frac{u^3}{3}\Big\vert_{-1}^1$$
3. Compute $F(b) - F(a)$ by plugging-in the upper limit (b) and substracting from it F plugged-in the lower limit (a).	**3.** $$[\frac{(1)^6}{6} - \frac{(1)^4}{4} + \frac{(1)^3}{3}] - [\frac{(-1)^6}{6} - \frac{(-1)^4}{4} + \frac{(-1)^3}{3}] = \frac{2}{3} \quad .$$

Problem Type 5.3.2 : Evaluate the (definite) integral

$$\int_a^b f(Var)d\,Var$$

where there are reciprocals and square-root signs inside the integral

Example Problem 5.3.2: Evaluate the (definite) integral

$$\int_1^4 \left(\sqrt{x} + \frac{1}{x^2} + \frac{3}{\sqrt{x}} \right) d\,x$$

Steps	**Example**
1. Convert everything to power notation. Remember $\sqrt{w} = w^{1/2}$ and $1/w^n = w^{-n}$.	**1.** $\int_1^4 \sqrt{x} + \frac{1}{x^2} + \frac{3}{\sqrt{x}} d\,x$ $= \int_1^4 (x^{1/2} + x^{-2} + 3x^{-1/2}) d\,x$
2. Find the antiderivative piece-by-piece using the famous rule $$\int x^n = \frac{x^{n+1}}{n+1}$$ (provided $n \neq -1$). Clean up. Get it to the form $$F(x)\Big\vert_a^b$$	**2.** $= \int_1^4 (x^{1/2} + x^{-2} + 3x^{-1/2}) d\,x$ $= \left(\frac{x^{3/2}}{3/2} + \frac{x^{-1}}{-1} + 3\frac{x^{1/2}}{1/2} \right) \Big\vert_1^4$ $= \left((2/3)x^{3/2} - \frac{1}{x} + 6\sqrt{x} \right) \Big\vert_1^4$
3. Compute $F(b) - F(a)$ by plugging-in the upper limit (b) and substracting from it F plugged-in the lower limit (a).	**3.** $= ((2/3)\cdot 4^{3/2} - \frac{1}{4} + 6\sqrt{4}) - ((2/3)\cdot 1^{3/2} - \frac{1}{1} + 6\sqrt{1})$ $= ((16/3) - \frac{1}{4} + 12) - ((2/3) - \frac{1}{1} + 6)$ $16/3 - 1/4 + 12 - (2/3) + 1 - 6 = \frac{137}{12}$.

Ans.: $\frac{137}{12}$.