

NAME:.....

Section:.....

MATH 151(07-09), Dr. Z. , **First Midterm**, Mon., Oct. 13, 2008.

1. (12 points) Find  $f'(4)$ , if

$$f(x) = \frac{\sqrt{x}}{2}$$

using the **definition** of the derivative. [No Credit for other methods].

2. (12 points) Find the equation of the tangent line to the curve

$$x^3 + 3xy + y^3 + y = 16 \quad ,$$

at the point  $(2, 1)$ .

3.

(a) (6 points) Show that the equation  $x^3 - 2 = 0$  has a solution in the open interval  $0 < x < 2$ .

(b) (6 points) Use the **formal definition** of the limit to prove that

$$\lim_{x \rightarrow -2} -2x - 3 = 1 \quad .$$

4. (16 points ([4 pts each]) Find the derivative  $f'(x)$  if:

(a)  $f(x) = \frac{x+2}{x^2+5}$

(b)  $f(x) = x^3 \sin x + 2x$

(c)  $f(x) = \frac{2+e^x}{1+2e^x}$

(d)  $f(x) = xe^{x^2}$

5.

(a) (6 points) If  $f(x) = e^{x^2} + x^2$ , find the second derivative  $f''(x)$ .

(b) (6 points) If the law of motion is  $s = t^7 + t$ , find the displacement (position), velocity, and acceleration at  $t = -1$ . Is it moving forward or backwards then? Is it speeding up or slowing down then?

6. (12 points [3 pts each]) Find the limits

$$(a) \quad \lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x + 1}$$

$$(b) \quad \lim_{x \rightarrow 0} \frac{x}{\sqrt{9 + x} - 3}$$

$$(c) \quad \lim_{x \rightarrow 0} \frac{3 - 3 \cos x + \sin x}{x}$$

$$(d) \quad \lim_{x \rightarrow 0} \frac{16 \sin^5 x}{\sin^2 2x \sin^3 x}$$

7. (12 points) Find the values of the constants  $a$  and  $b$  that will make the function

$$f(x) = \begin{cases} x^2, & \text{if } x < 1; \\ ax + b, & \text{if } x \geq 1. \end{cases}$$

differentiable everywhere.

8. (12 points) Find the point on the curve

$$y = -\cos x \quad ,$$

with  $0 < x < \pi$ , where the tangent line is parallel to the line  $y = \frac{x}{2} + 4$ . Then find the equation of that tangent line.